Memo: some pitfalls of the wide field astrometry

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1 Problem statement

It is not uncommon to find a source far a way from its a priori position. For instance, source 1843–001 was found $299''.73$ off its a priori position used by the correlator in VLBA experiment bf071b (VCS2) on 2002.05.14. In this memo I discuss complications that arise in processing VLBI experiments with an ultra-wide field.

There are four effects that are to be taken into consideration: 1) reduction of the fringe amplitude; 2) setting correct search window; 3) taking into account the second derivative of source position errors on source coordinate; 4) taking into account the quadratic curvature of the fringe phase that is due to the third mixed derivative of time delay on time and on source coordinates. The first three effects are well known. I am not sure the last effects was ever mentioned.

2 Reduction of fringe amplitude

A correlator provides the cross-correlation spectrum at discrete samples, called in VLBI slang uv-points, averaged over frequency channels with the spectral resolution Δf (f is a cyclic frequency) and over accumulation period duration of Δt. The residual delay and delay rate over an individual spectrum sample causes the decorrelation of the fringe amplitude. The average output amplitude L for a continuum signal with the input amplitude 1.0 will be

$$\begin{aligned} L = \; & \left| \frac{1}{\Delta f \Delta t} \int\limits_{f-\Delta f/2}^{f+\Delta f/2} \int\limits_{t-\Delta t/2}^{t+\Delta t/2} \cos(2\pi\tau f)\cos(2\pi f_o \dot{\tau} t)\; df dt \right| = \\ & \cos(2\pi\tau f)\cos(2\pi f_o \dot{\tau} t) \left| \frac{\sin \pi\tau\Delta f}{\pi\tau\Delta f} \, \frac{\sin \pi f_o \dot{\tau}\Delta t}{\pi f_o \dot{\tau} \Delta t} \right| . \end{aligned} \tag{1}$$

The coarse fringe search is performed on a grid of delays τ and delay rates $\dot{\tau}$. The natural spacings of the grid are $1/\Delta f$ and $1/\Delta t$. This defines the search window $[-1/2\Delta f,\, 1/2\Delta f]$ for delay and $[-\frac{1}{2f_o\Delta t},\, \frac{1}{2f_0\Delta t}]$ for delay rate. The coherence loss L is a product of two sinc functions (the cosine terms insignificantly differ from 1). Both sinc terms contribute for both longitudinal and latitudinal baselines, only the first term contribute for a latitudinal baseline. The amplitude loss reaches $2/\pi = 0.637$ at the edge of the natural search window when one sinc factor plays a role and $4/\pi^2 = 0.405$ when both factors contribute.

The amplitude loss grows rapidly when the fringe search takes place beyond the natural window. The coherence drops from 0.637 to 0 when delay goes from $1/2\Delta f$ to $1/\Delta f$, reaches maximum $2/3\pi = 0.212$ at $3/2\Delta f$ and again drops to 0 at $2/\Delta f$.

This effect has two consequences: 1) weak sources are not detected at delays and delay rates close to boundaries of the natural search window and beyond them; 2) the fringe amplitude is underestimated when delays and delay rates are comparable with the range of the natural search window. $\mathcal{PIMA}$ has an option to correct the fringe amplitude for decorrelation loss, i.e. to divide the amplitude by L. Do not know, whether AIPS has it.

3 Setting correct search window

As we showed, the fringe amplitude drops at delays and delay rates close to the search window boundary. Beyond the search window weak sources will not be detected, but a strong source still may be detected. However, in that case the coarse search procedure will end up with the delay folded to the opposite part of the search window due to folding of the Fourier transform of the fringe spectrum. Let us consider an example: VLBA experiment bc191a, source 0802–010, baseline HN-VLBA/MK-VLBA. The natural search window for delay is $[-4.0, 4.0]$ mks. Expected group delay is $-4.779486 \cdot 10^{-6}$ sec, but the fringe search procedure gave $3.220556 \cdot 10^{-6}$, close to $8 \cdot 10^{-6} + -4.779486 \cdot 10^{-6}$. Such points are easily identified by analyzing residuals: their values are the full window length or a multiple window width. For example, the point in the example above had residual 8000 ns.

$\mathcal{PIMA}$ allows to search beyond the natural window if the region is specified with accuracy at least 1/4 of the natural window. The algorithm merely resolves the integer ambiguity by modulo $1/Deltaf$ for delay and/or $1/(f_o\,\Delta t)$ for delay rate at the center of the specified window and then adds back subtracted value of delay and/or delay rate to the value found by the fringe search procedure. In the example above re-fringing with window centered around -4.7 mks with width ± 1.0 mks gave correct value of group delay $-4.779448 \cdot 10^{-6}$.

Apparently, AIPS does not have such a feature.

4 Taking into account the second derivative over source positions

The maximum magnitude of the second derivative of path delay over right ascension or declination is of the order $\frac{|b|}{c}$, i.e. 0.02 sec for the baseline with lengths of the Earth's radius. Considering typical errors of group delay determination as $2 \cdot 10^{-11}$ sec, the source position error of $4.5 \cdot 10^{-5}$ rad ($9''$) will lead to the contribution of the second derivative over source position at 1σ level ($1/2 \times (4.5 \cdot 10^{-5})^2 \times 0.02 \approx 2 \cdot 10^{-11}$).

Therefore, several iterations of astrometric solution and a priori position source update should be run when a priori source position has errors exceeding $5''$. This is relatively easy procedure. No re-fringing is needed for mitigation of this effect.

5 taking into account the quadratic curvature of the fringe phase.

We consider the fringe phase over a scan varies linearly with both time and frequency in the fringe search procedure. The phase at accumulation period i and frequency channel j is expressed as

$$\varphi_{ij} = \omega_o \tau_p + \omega_o (t_i - t_o) \dot{\tau_p} + (\omega_j - \omega_o)\, \tau_g + (\omega_j - \omega_o)\, (t_i - t_o)\, \dot{\tau_g} \tag{2}$$

where ω is an angular frequency. τ_p and τ_g are phase delay and group delay. However, when the a priori position is too far away and the scan time is long, the third mixed delay derivatives,

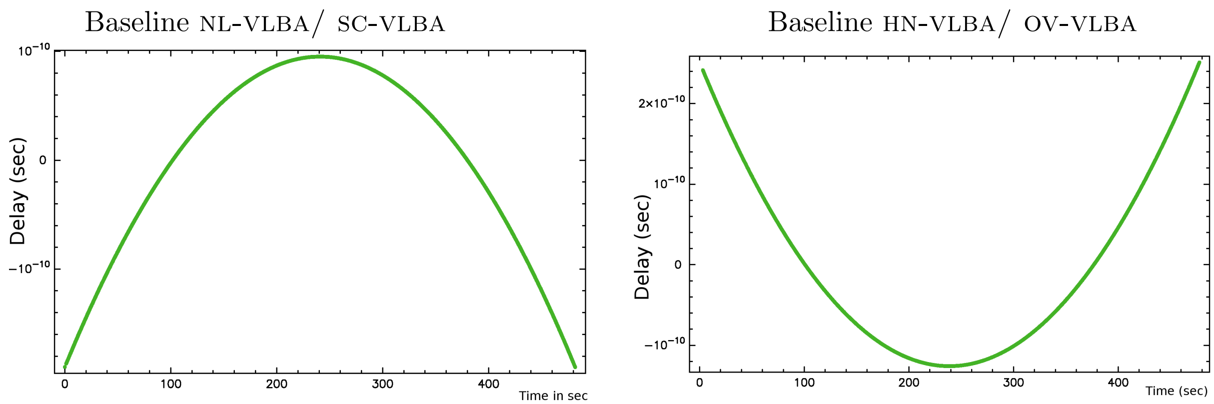

Figure 1: Contribution of a product of the third mixed derivative of time delay over time and source coordinate and the source displacement with respect to the a priori position used for correlation at experiment bc191a. Source 0802–010 was detected $40''.8$ away from its a priori position.

namely $\dfrac{\partial^3 \tau}{\partial^2 t \partial \alpha}$ and $\dfrac{\partial^3 \tau}{\partial^2 t \partial \delta}$, may result in a significant quadratic term of a dependence of the fringe phase on time over the scan time period.

If to ignore gravitational refraction and propagation effects, the time delay is expressed up to terms $O(1/c^3)$ as:

$$\tau = \frac{1}{c}\,\vec{b}\cdot\underline{\vec{\mathbf{S}}}\,\frac{1}{1+\dfrac{1}{c}(\vec{\mathbf{V}}_\oplus+\dot{\vec{\mathbf{r}}}_2)\cdot\underline{\vec{\mathbf{S}}}} \;+\; \frac{1}{c^2}(\vec{\mathbf{V}}_\oplus\cdot\vec{b}) \;+\; O(1/c^3). \qquad (3)$$

Differentiating this expression over time twice, we get an expression for $\ddot{\tau}$:

$$\ddot{\tau} = \frac{1}{c}\,\frac{\ddot{\vec{\mathbf{b}}}\cdot\underline{\vec{\mathbf{S}}}}{1+\dfrac{1}{c}(\vec{\mathbf{V}}_\oplus+\dot{\vec{\mathbf{r}}}_2)\cdot\underline{\vec{\mathbf{S}}}} \;-\; \frac{2}{c^2}\,\frac{(\dot{\vec{\mathbf{V}}}_\oplus+\ddot{\vec{\mathbf{r}}}_2)\cdot\underline{\vec{\mathbf{S}}}}{\left(1+\dfrac{1}{c}(\vec{\mathbf{V}}_\oplus+\dot{\vec{\mathbf{r}}}_2)\cdot\underline{\vec{\mathbf{S}}}\right)^2} \;+\; \frac{2}{c^2}\dot{\vec{\mathbf{V}}}_\oplus\cdot\dot{\vec{\mathbf{b}}} \;-\; \frac{2}{c^2}\vec{\mathbf{V}}_\oplus\cdot\ddot{\vec{\mathbf{b}}} \;+\; O(1/c^3). \qquad (4)$$

The maximum value of $\ddot{\tau}$ for a baseline with the Earth's radius is $\frac{1}{c}R_\oplus\Omega^2_\oplus$, or about $1\cdot 10^{-10}$ 1/sec. The maximum value of $\dfrac{\partial^3\tau}{\partial^2 t\partial\delta}$ is about the same as $\ddot{\tau}$. Let us consider a case when a source observed at 8.4 GHz in a scan of 8 minutes long is $5\cdot 10^{-5}$ rad $(10'')$ off at a baseline with the length of the Earth's radius. Then the maximum contribution of the phase curvature over the scan will be $2\pi f\times\frac{1}{c}R_\oplus\Omega^2_\oplus(t/2)^2/2\approx 7.6$ rad!

Figures 1 illustrates the contribution of $\dfrac{\partial^3\tau}{\partial^2 t\partial\underline{\vec{\mathbf{S}}}}\Delta\underline{\vec{\mathbf{S}}}$ term to path delay for 0802–010 with the a priori error of $40''.8$ observed at NL-VLBA/ SC-VLBA and HN-VLBA/ OV-VLBA at 8.4 GHz.

Such contribution is in not negligible. Figures 2–3 show result of fringe search of 0802–010. Fringe phase made several wraps over 480 seconds. The presence of the quadratic term that was not taken into account in the fringe search procedure caused significant decorrelation of the coherently averaged cross-spectrum: a factor of 3–5. It should be noted that this decorrelation is applied on top of decorrelation of individual cross-spectrum samples situated at the edges of the search window.

The contribution of the 3rd mixed derivative can be removed in an iterative data analysis procedure. At the first step, source coordinates are determined using observations at short

Baseline NL-VLBA/ SC-VLBA | Baseline OV-VLBA/ SC-VLBA

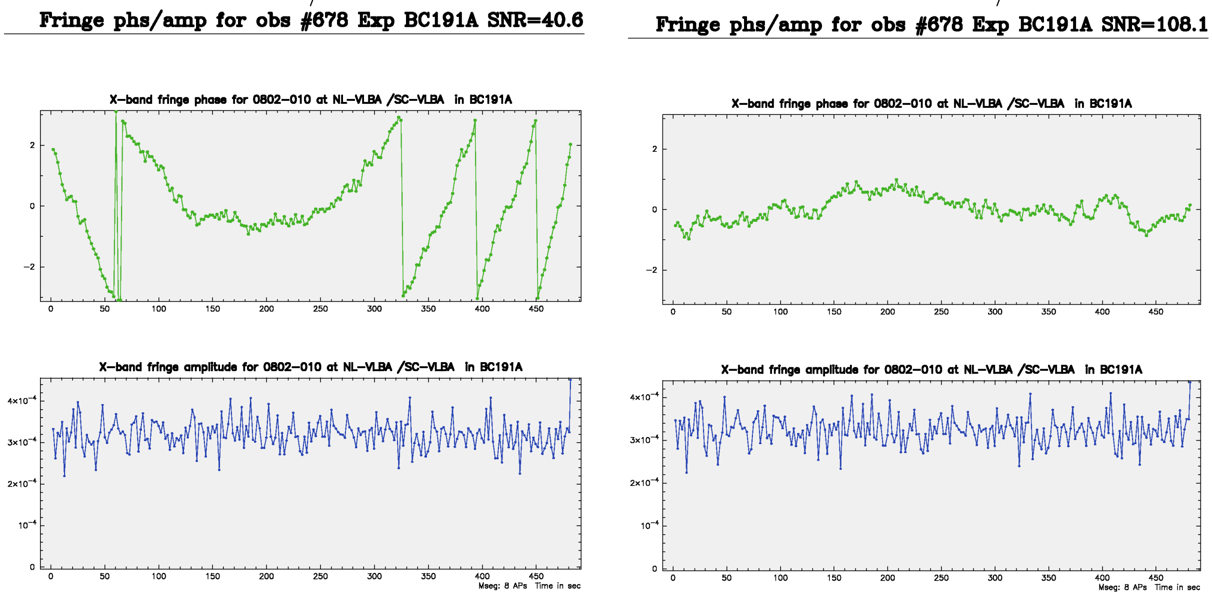

Figure 2: Fringe phase and fringe amplitude as a function of time of source 0802–010 in VLBA experiment bc191a without (left) and with (right) correction for $\dfrac{\partial^3 \tau}{\partial^2 t \partial \underline{\vec{S}}} \, \Delta \underline{\vec{S}}$ term. Source 0802–010 was detected $40''.8$ away from its a priori position.

Baseline NL-VLBA/ SC-VLBA | Baseline OV-VLBA/ SC-VLBA

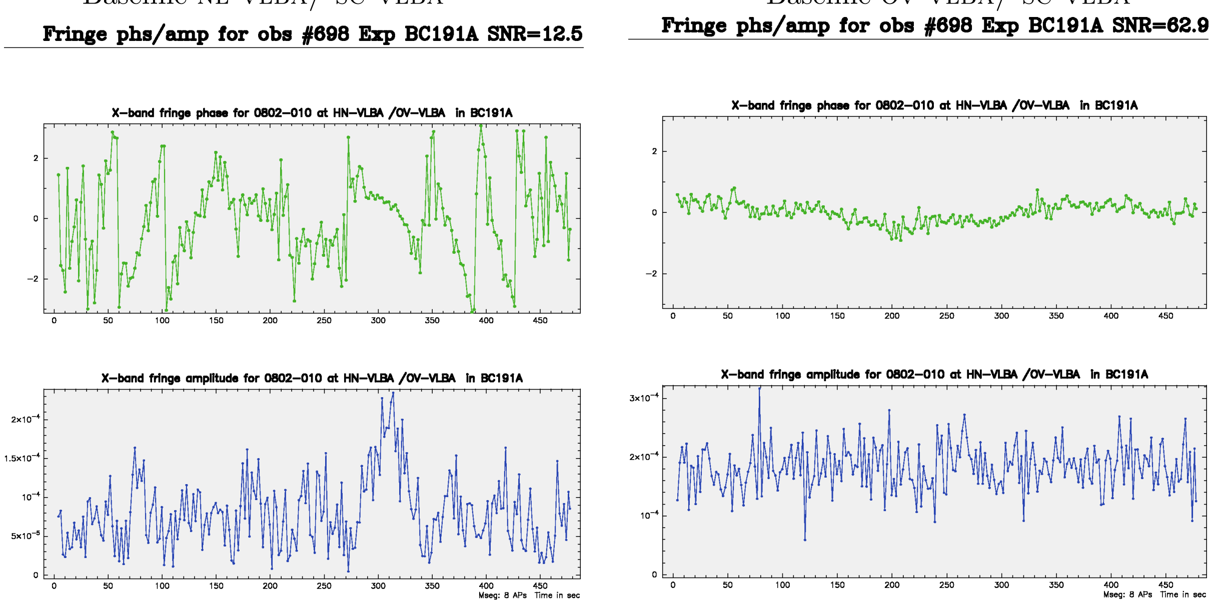

Figure 3: Fringe phase and fringe amplitude as a function of time after fringe search of source 0802–010 in VLBA experiment bc191a without (left) and with (right) correction for $\dfrac{\partial^3 \tau}{\partial^2 t \partial \underline{\vec{S}}} \, \Delta \underline{\vec{S}}$ term. Source 0802–010 was detected $40''.8$ away from its a priori position.

baselines. Since the derivative is proportional to the baseline length, its contribution is low at short baselines. At the second step, the fringe search is repeated and term $\dfrac{\partial^3 \tau}{\partial^2 t \partial \underline{\vec{S}}} \Delta \underline{\vec{S}}$ is computed using $\Delta \underline{\vec{S}}$ — the correction to source position with respect to the a priori used during correlation. This contribution is computed for each accumulation period and a second polynomial with respect to fringe reference time is computed: $\dfrac{\partial^3 \tau}{\partial^2 t \partial \underline{\vec{S}}} \Delta \underline{\vec{S}} = \tau_c + (t_i - t_o)\dot{\tau}_c + \frac{1}{2}(t_i - t_o)^2 \ddot{\tau} c$. The zero-th order contribution, τ_c — the mean value, is discarded. Then the fringe phases of the initial cross-correlation spectrum are corrected for the presence of the first and second degree constituent in the fitted polynomial:

$$\varphi_{ij} = \varphi_{ij} \; - \; (\omega_j - \omega_o) \; (t_i - t_o) \; \dot{\tau}_c \; - \; \frac{1}{2} \omega_j \; (t_i - t_o)^2 \; \ddot{\tau}_c^2 \tag{5}$$

Corrected cross-correlation spectrum used by the fringe search procedure. Finally, the coefficient τ_1 is added to the estimated $\dot{\tau}_g$.

This procure is implemented in $\mathcal{PIMA}$. Two source catalogues are updated, the first catalogue is extracted from input data files in FITS-IDI format. That catalogue keeps source positions used by the correlator. The second catalogue us used by $\mathcal{PIMA}$ for computing a priori path delay. If the a priori position exceeds $3 \cdot 10^{-6}$ rad ($0''.6$) and the scan length is over 60 s, then path delay is computed for each accumulation period for two sources positions: one used by the correlator and another from the external catalogue, i.e. taken from the previous preliminary solution. The second order polynomial is fitted to the differences. Estimates $\dot{\tau}_c$ and $\ddot{\tau}_c$ are used for phase correction prior the coarse fringe search using formula 5.

Results of this procedure are shown at the right parts of figures 2–3. Improvement is very significant.

I am not aware of whether AIPS has such a procedure.

References

Thompson A. R., Moran J.M., Swenson, G.W. 2001 Larger Image Interferometry and Synthesis in Radio Astronomy, Willey Lmt.