# Combined analysis of VLBI and superconducting gravimeter data

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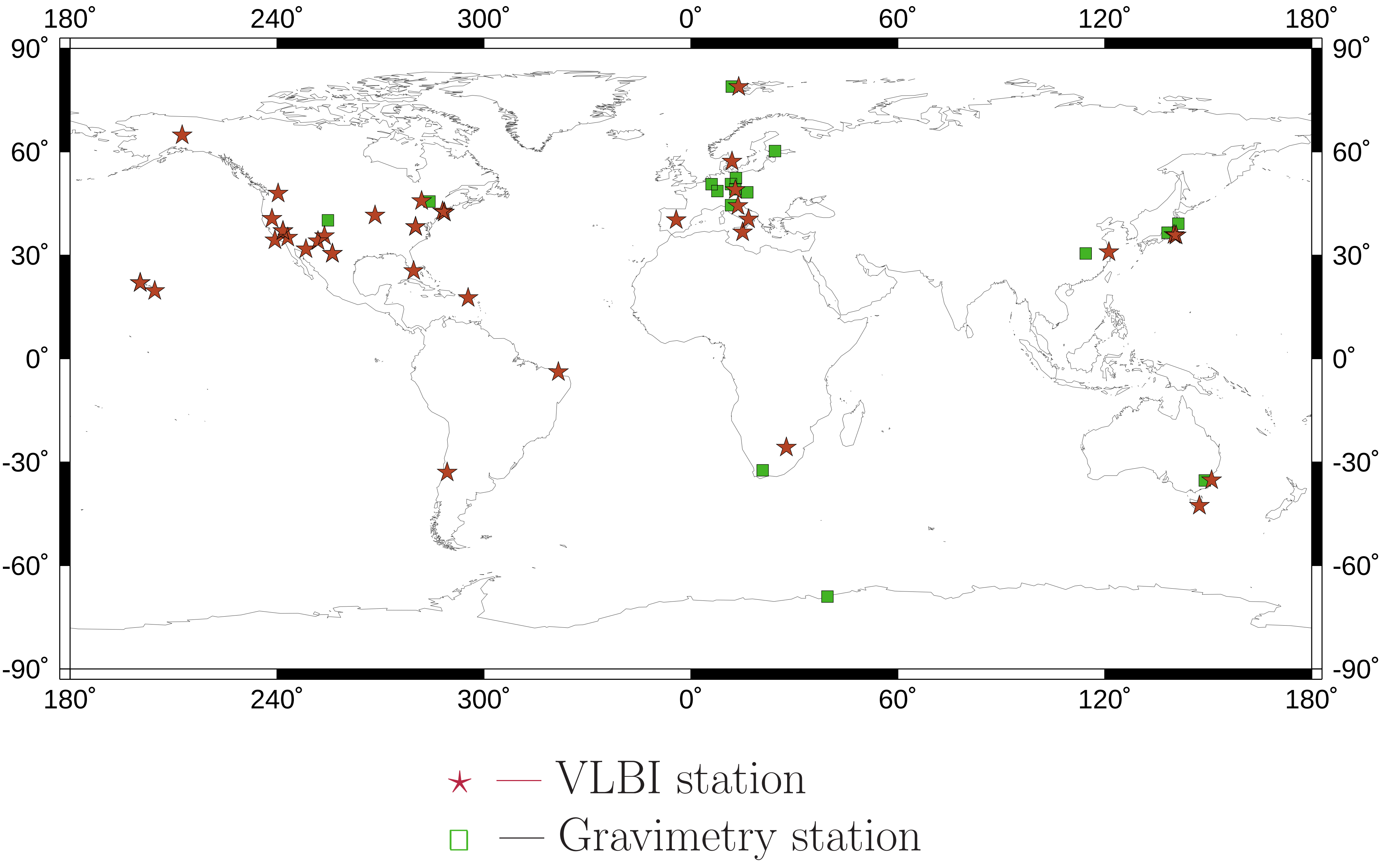
# Motivation:

- Displacement and gravity measurements are sensitive to different combinations of Love numbers.

- Displacement and gravity measurements are sensitive to different wavelength of loading: gravity is more sensitive to short wavelengths, displacements are sensitive to longer wavelengths.

- Displacement and gravity measurements have quite different systematic errors and are in a great extent independent. Therefore, can be used for verification of results.

# VLBI and Gravimetry stations



# VLBI data analysis

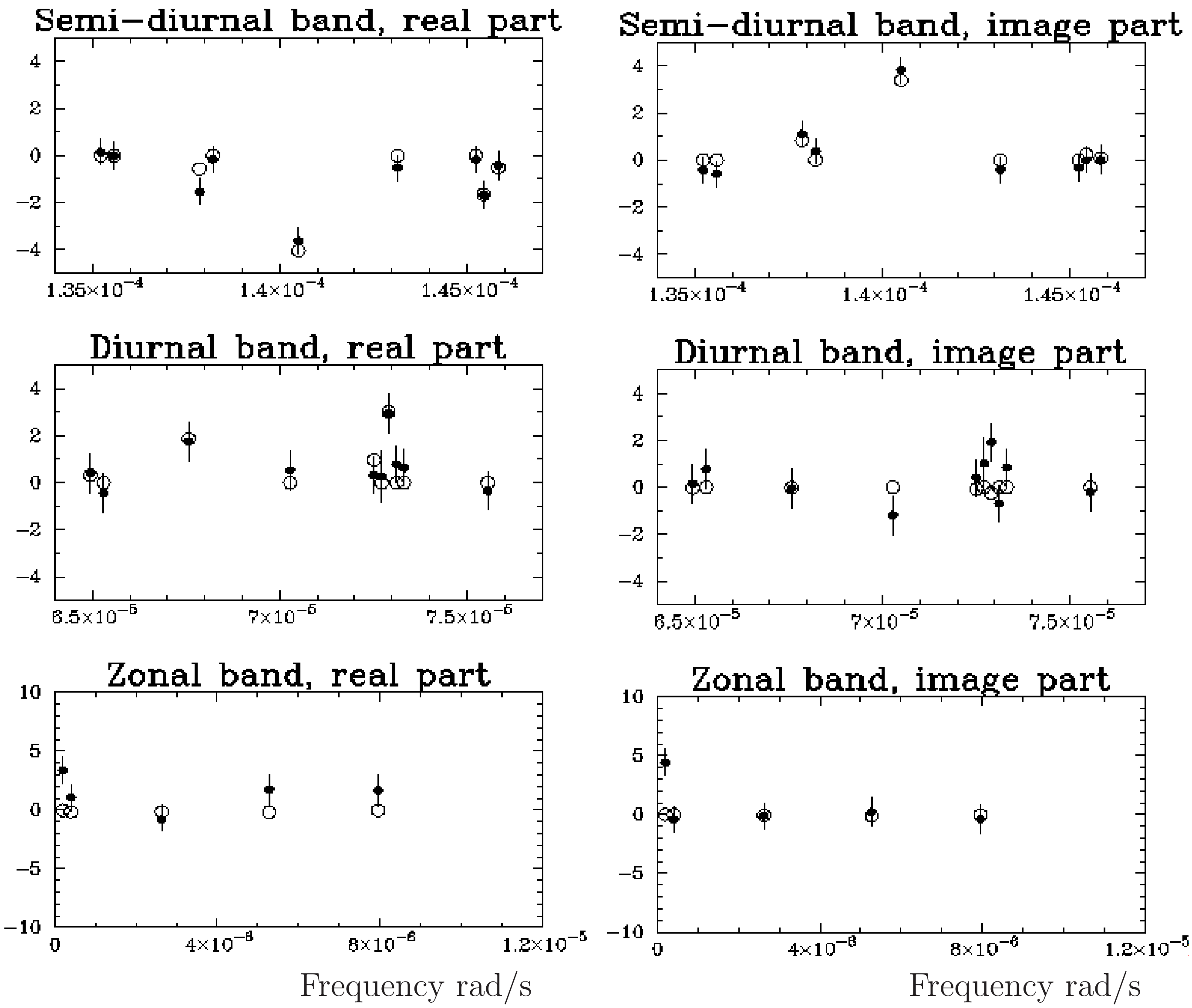
- **Data:** $3.7 \cdot 10^6$ time delays between 40 stations for 1980.0–2003.5.

- **Theoretical models:** post-IERS2003 solid Earth tides, pole tide; ocean, atmospheric and hydrology loadings; empirical model of harmonic EOP variations; IMF troposphere path delay model, etc.

- **Estimation Model:**
  - Harmonic position variations solution: targeted parameters: variations of positions of all sites at 37 tidal frequencies. *(No ocean loading was applied)*
  - Love numbers solution: targeted parameters: Love numbers $h$ and $l$, real and imaginary parts for 37 tidal frequencies. *(Ocean loading was applied)*

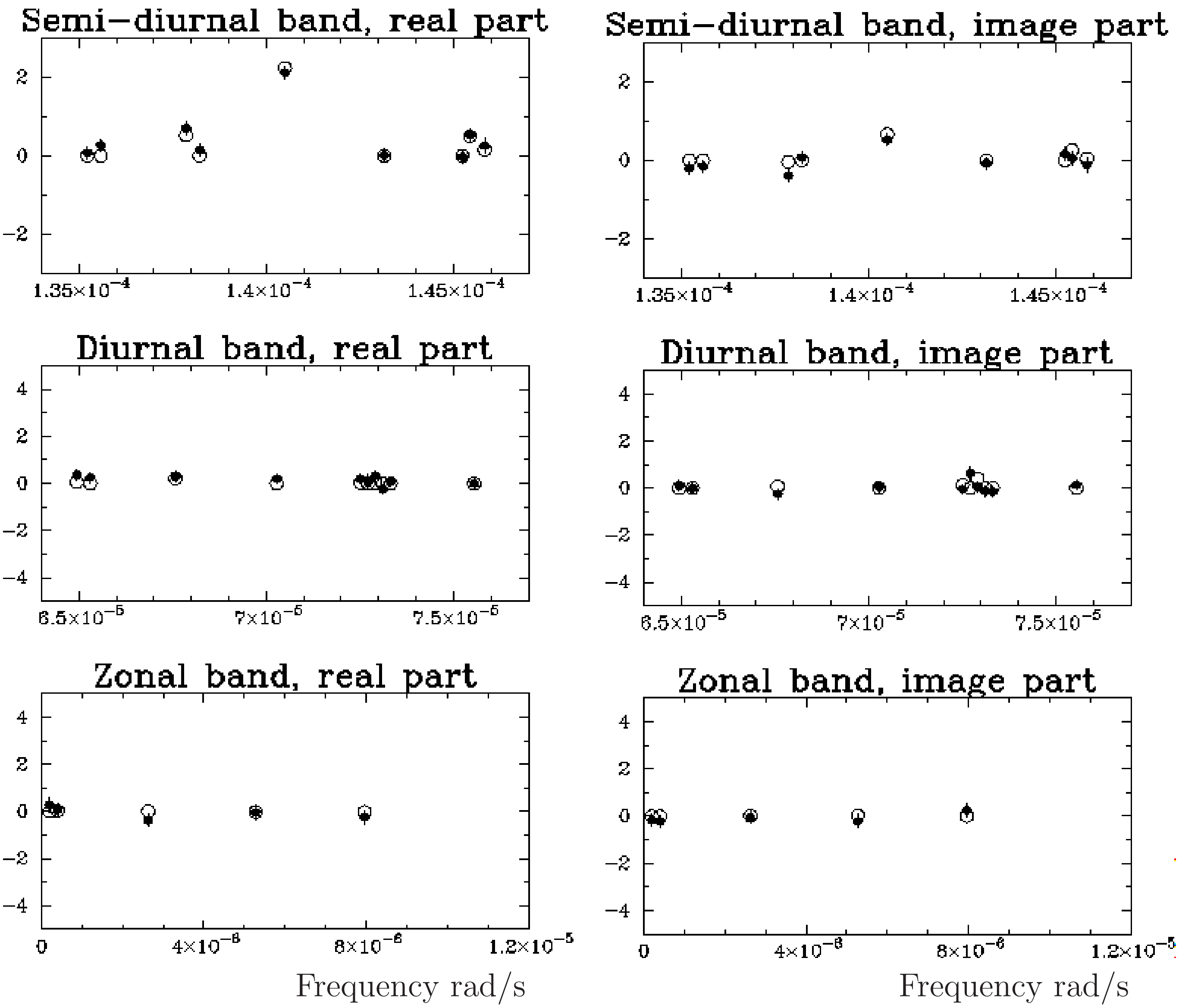
and $1.2 \cdot 10^6$ nuisance parameters.

Residual harmonic site position variation in mm
Station Algopark, Up direction




circles— Theoretical ocean loading contribution;
**disks**— Results of VLBI data analysis;

Residual harmonic site position variation in mm
Station Algopark, East direction




circles— Theoretical ocean loading contribution;
**disks**— Results of VLBI data analysis;

# VLBI residual displacements statistics

| Tide | Residual displacements | | $\chi^2$/ndg | |
|---|---|---|---|---|
| | vert. | horz. | vert. | horz. |
| $M_3$ | 0.5 mm | 0.2 mm | 0.6 | 1.1 |
| $S_2$ | 1.3 mm | 0.4 mm | 2.8 | 1.9 |
| $M_2$ | 1.0 mm | 0.4 mm | 1.3 | 2.0 |
| $N_2$ | 0.9 mm | 0.3 mm | 1.1 | 1.0 |
| $\psi_1$ | 1.1 mm | 0.4 mm | 0.9 | 1.2 |
| $K_1$ | 1.6 mm | 0.7 mm | 1.5 | 3.6 |
| $S_1$ | 2.8 mm | 0.7 mm | 2.8 | 2.2 |
| $O_1$ | 1.1 mm | 0.4 mm | 0.7 | 1.4 |
| Mf | 1.4 mm | 0.6 mm | 1.1 | 0.9 |
| Ssa | 2.1 mm | 0.8 mm | 1.9 | 1.1 |
| Sa | 4.0 mm | 1.3 mm | 6.3 | 2.6 |

(*All loading contributions have been applied*)

# Superconducting gravimetry data analysis

- **Data:** 1 minute data from 17 station. Filtered to 1 hour. Start date: 1987.0, end date: 2003.5. Average length: 4.5 years.
- **Theoretical models:**
  - Solid Earth tide model DDW99;
  - Contribution of pole tide;
  - Contribution of ocean loading: NAO99b model, equilibrium ocean model for pole tide;
  - Contribution of atmospheric pressure loading;
  - Contribution of hydrology loading.
- **Estimation models:**
  - $\delta$ factor for 37 targeted frequencies, real and imaginary parts;
  - $\delta$ factor at the frequencies where is not tides for evaluation of re-weighting parameters.

Software: Eterna.

# Gravity residual accelerations statistics

| Tide | Residual acc. | $\chi^2$/ndg |
|---|---|---|
| $M_3$ | 0.2 nm/$s^2$ | 2.6 |
| $S_2$ | 0.5 nm/$s^2$ | 16.9 |
| $M_2$ | 0.8 nm/$s^2$ | 54.3 |
| $N_2$ | 3.5 nm/$s^2$ | 621.7 |
| $\psi_1$ | 0.1 nm/$s^2$ | 0.8 |
| $K_1$ | 1.0 nm/$s^2$ | 112.6 |
| $S_1$ | 3.7 nm/$s^2$ | 552.4 |
| $O_1$ | 1.1 nm/$s^2$ | 98.1 |
| Mf | 0.6 nm/$s^2$ | 32.7 |
| Ssa | 2.3 nm/$s^2$ | 332.1 |
| Sa | 4.3 nm/$s^2$ | 3689.1 |

*(All loading contributions have been applied)*

# Error analysis

Approach: to find lower and upper limits for errors in the estimates.

- **lower error limit**.
  *Assumptions: Covariance matrix of errors is known, perhaps up to some factor. Noise is white.*
  - Estimation of a measurement error.
  - Errors propagation law.
  - Estimation Love numbers and $\delta$ factors at the frequencies where there is no signal.
  - Errors rescaling.
- **upper error limit**.
  *Assumptions: Errors over all stations have zero mean. There is no signal in residuals, only noise.*
  - Evaluate residual displacements and gravity acceleration at each frequency and each station individually.
  - Evaluate for each frequency rms of residuals over all stations.
  - Use these rms as a measure of in-band errors for computing the errors for Love numbers and $\delta$ factors.

## Properties of two error estimates:

- **low error limit** is always less than the **upper error limit**
- if the number of stations $\longrightarrow \infty$,
  time of observation $\longrightarrow \infty$,
  then **low error limit** $\longrightarrow 0$.
- if the number of stations $\longrightarrow \infty$,
  time of observation $\longrightarrow \infty$,
  then **upper error limit** $\longrightarrow \lim > 0$.
  The more stations, the more observations, the closer the estimate of the **upper error limit** approach to that limit.

## Relative accuracy of different type of parameters in %

| Tide | $\Delta h/h$ | $\Delta l/l$ | $\Delta\delta/(\delta-1)$ | $\Delta k/k$ |
|---|---|---|---|---|
| Ssa | 4 — 25 | 4 — 8 | 6 — 40 | 3 — 40 |
| Mf | 1 — 6 | 1 — 2 | 2.5 — 5 | 1.6 — 9 |
| $K_1$ | 0.2 — 0.6 | 0.4 — 0.7 | 0.2 — 1.2 | 0.2 — 0.8 |
| $O_1$ | 0.2 — 0.5 | 0.2 — 0.9 | 0.3 — 1.8 | 0.2 — 1.1 |
| $M_2$ | 0.07 — 0.3 | 0.1 — 0.3 | 0.2 — 1.2 | 0.1 — 0.7 |
| $S_2$ | 0.2 — 1.1 | 0.2 — 0.7 | 0.4 — 1.2 | 0.3 — 0.5 |
| $M_3$ | 15 — 40 | 20 — 70 | 30 — 30 | 22 — 65 |

A range `lower error` — `upper error` is shown.

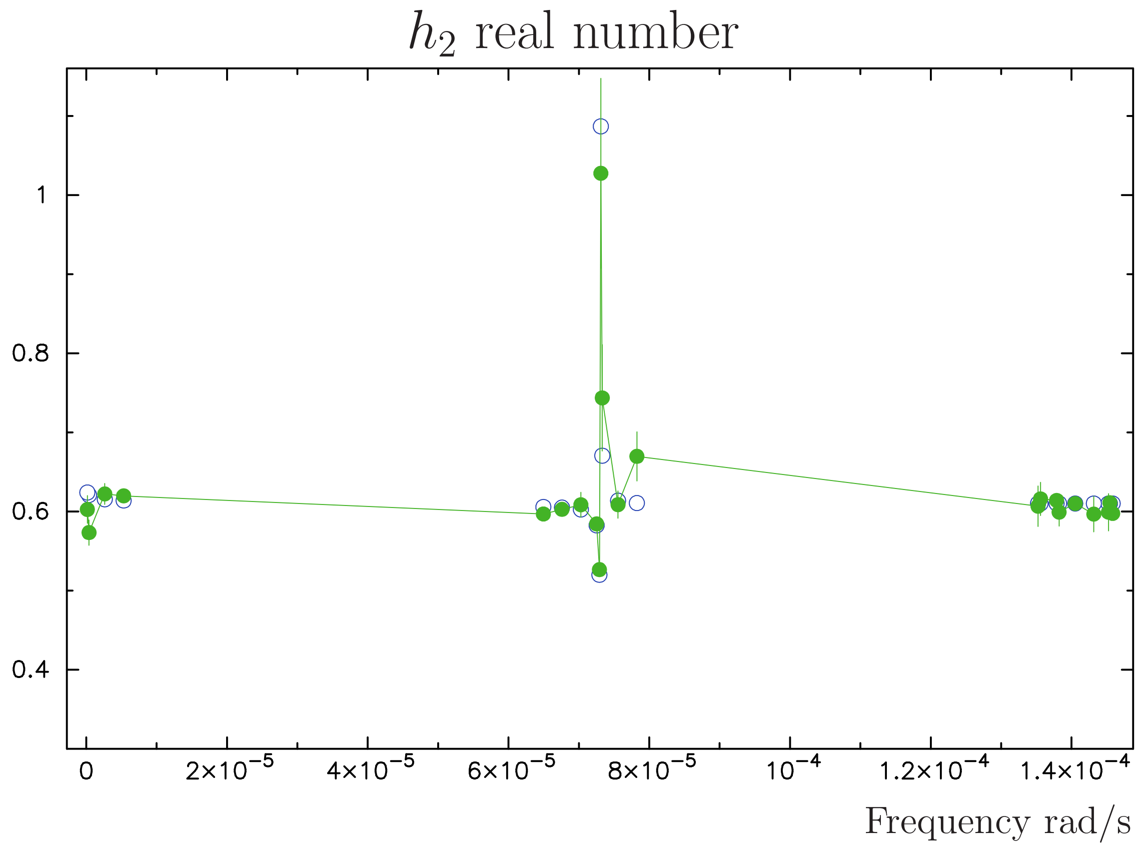
Love number $k$ is computed from a combination of VLBI and gravity results for the spherical Earth:

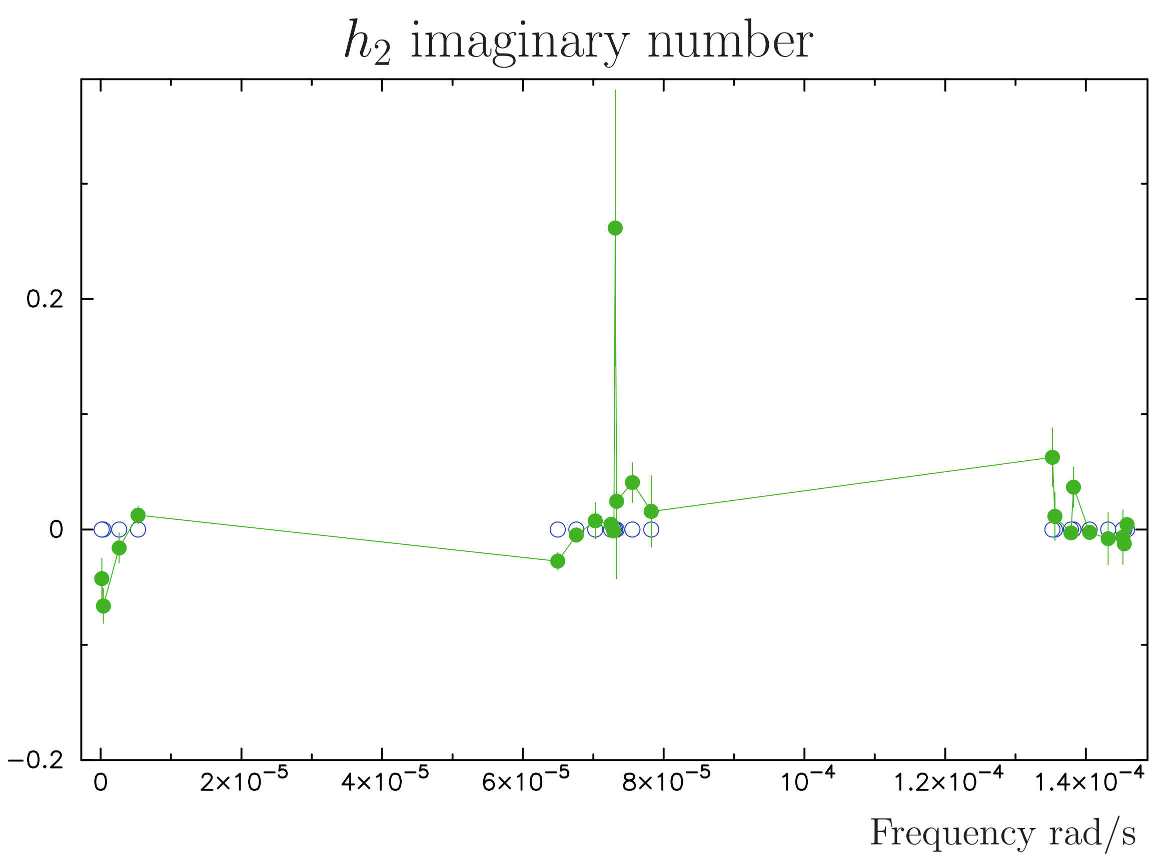
$$\delta_2 = 1 + h_2 - \frac{3}{2}k_2$$

$$\delta_3 = 1 + \frac{2}{3}h_3 - \frac{4}{3}k_3$$

However, only 15% of $\delta_2$ and 7% of $\delta_3$ are sensitive to Earth's non-rigidity.

# Love number $h_2$ from VLBI estimation



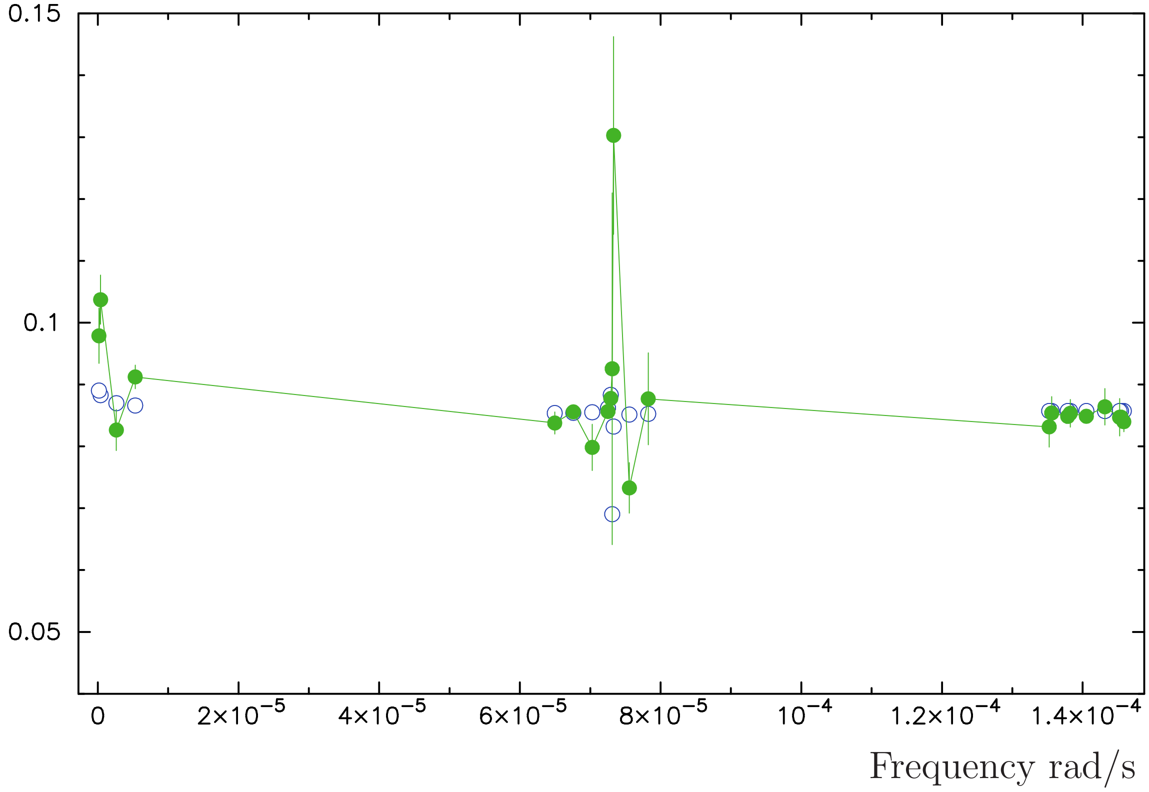




● — VLBI estimates

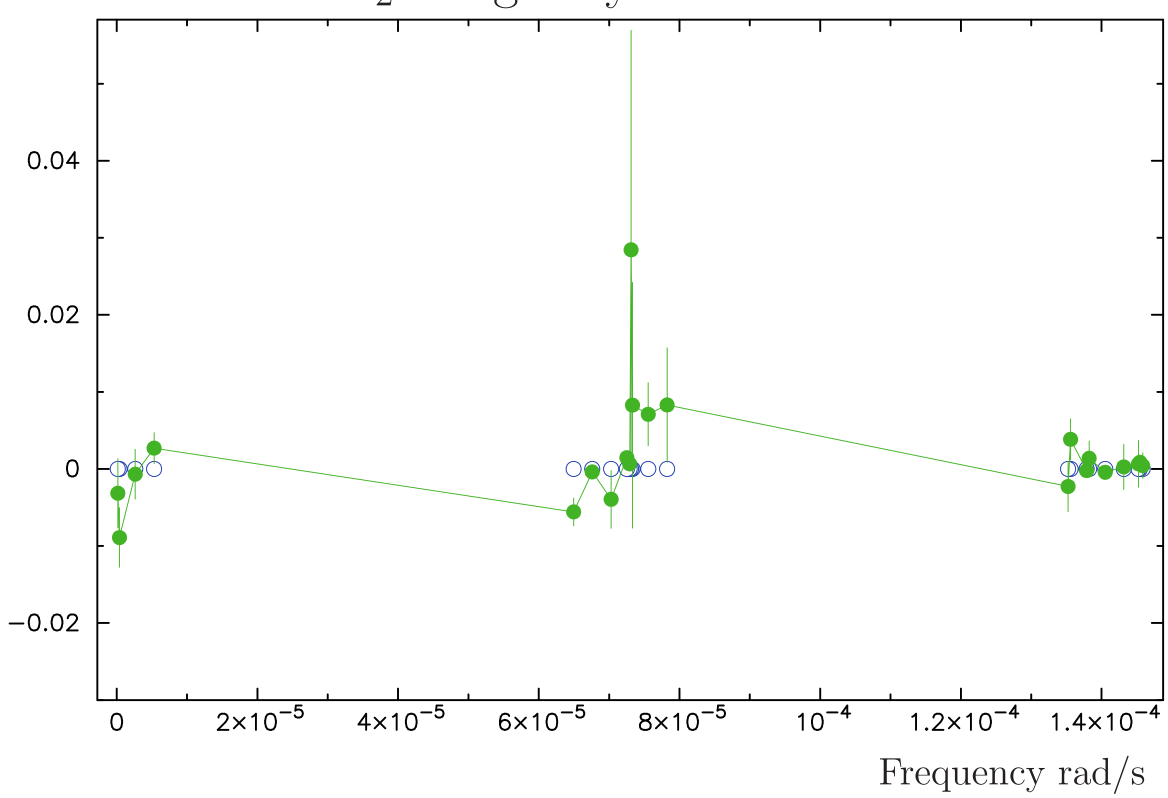
○ — Theory (DDW99in) predictions

# Love number $l_2$ from VLBI estimation

$l_2$ real number




$l_2$ imaginary number

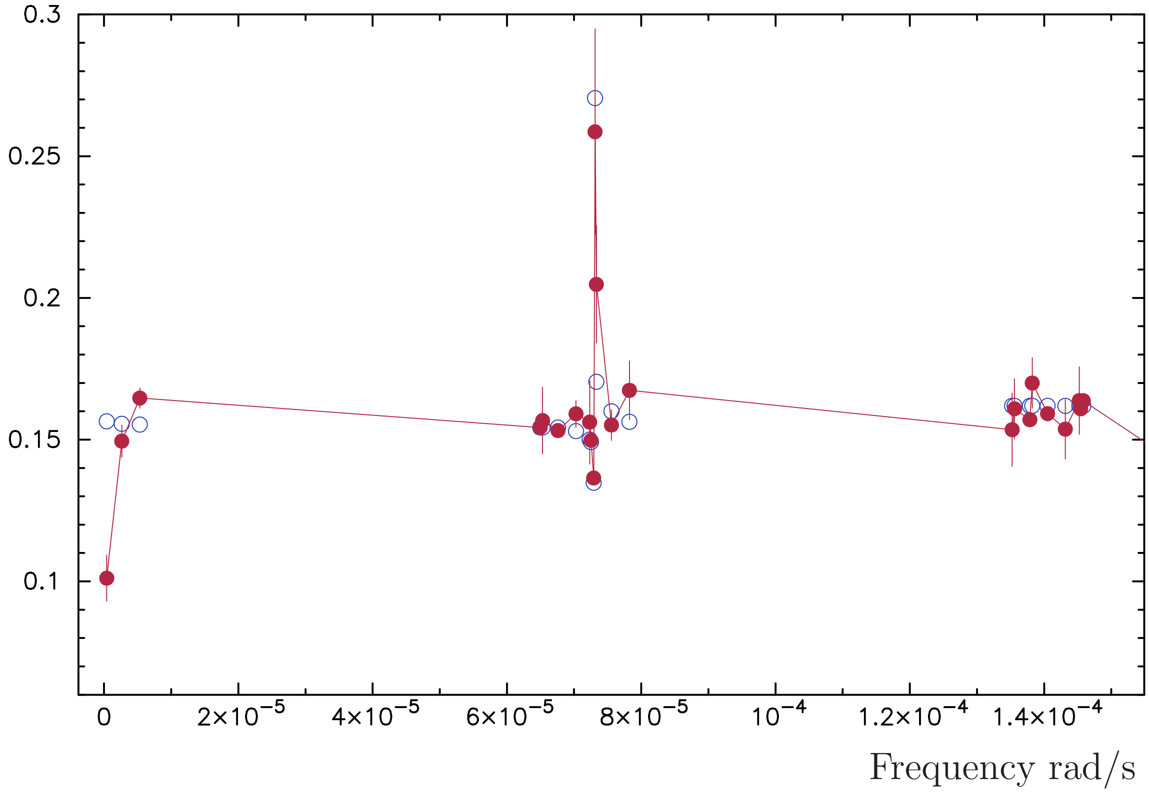



● — VLBI estimates

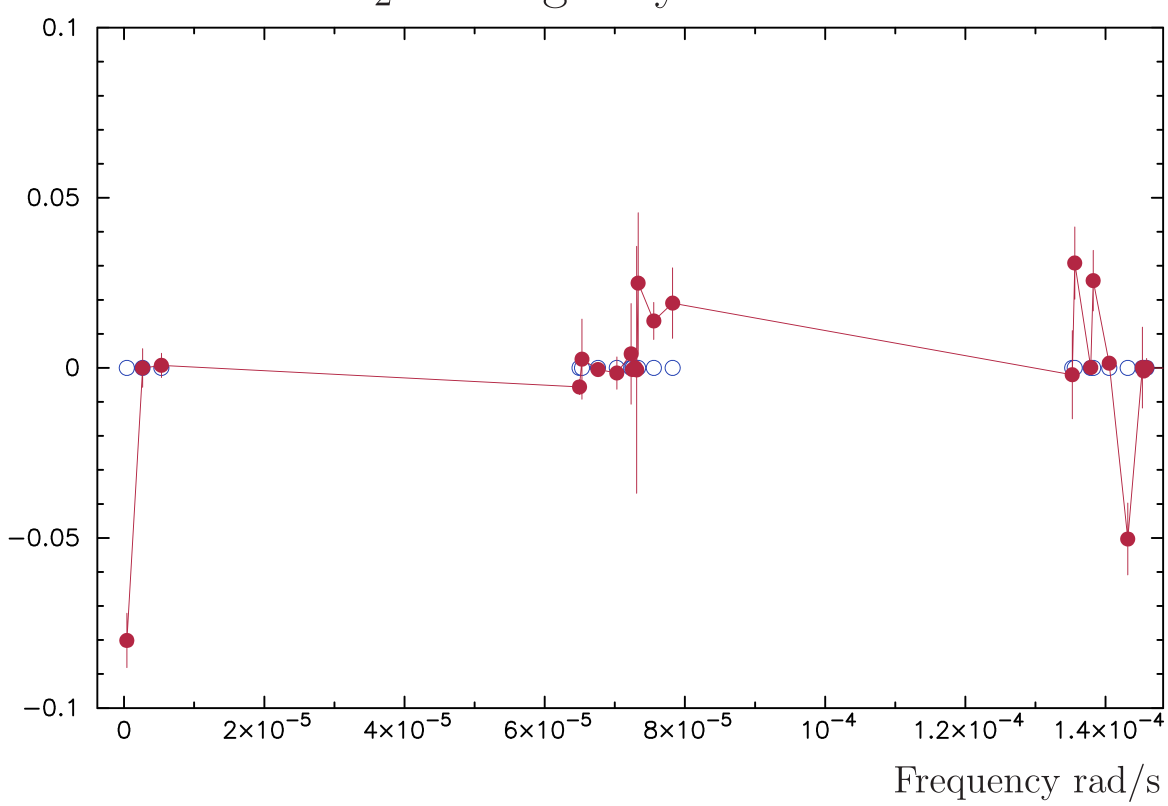
○ — Theory (DDW99in) predictions

# $\delta_2$ factors from superconducting gravimetry

$\delta_2$–1 real factor



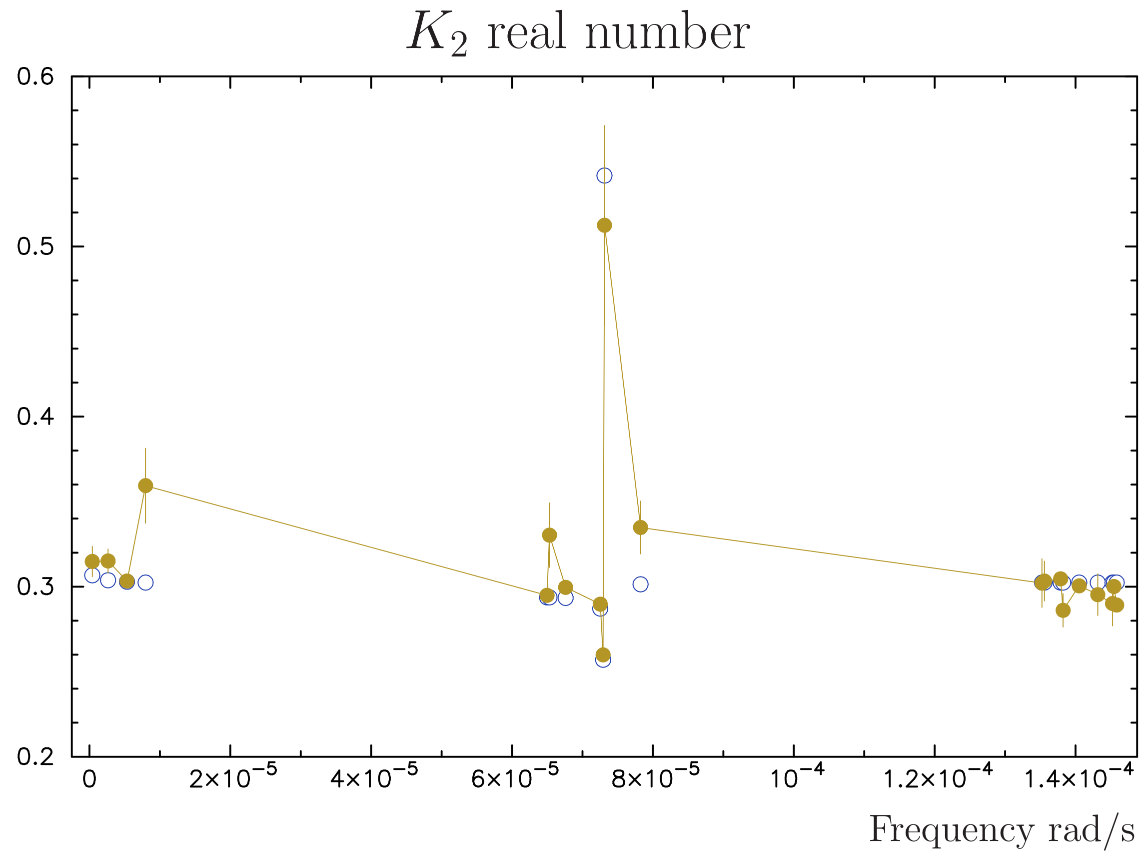

$\delta_2$–1 imaginary factor

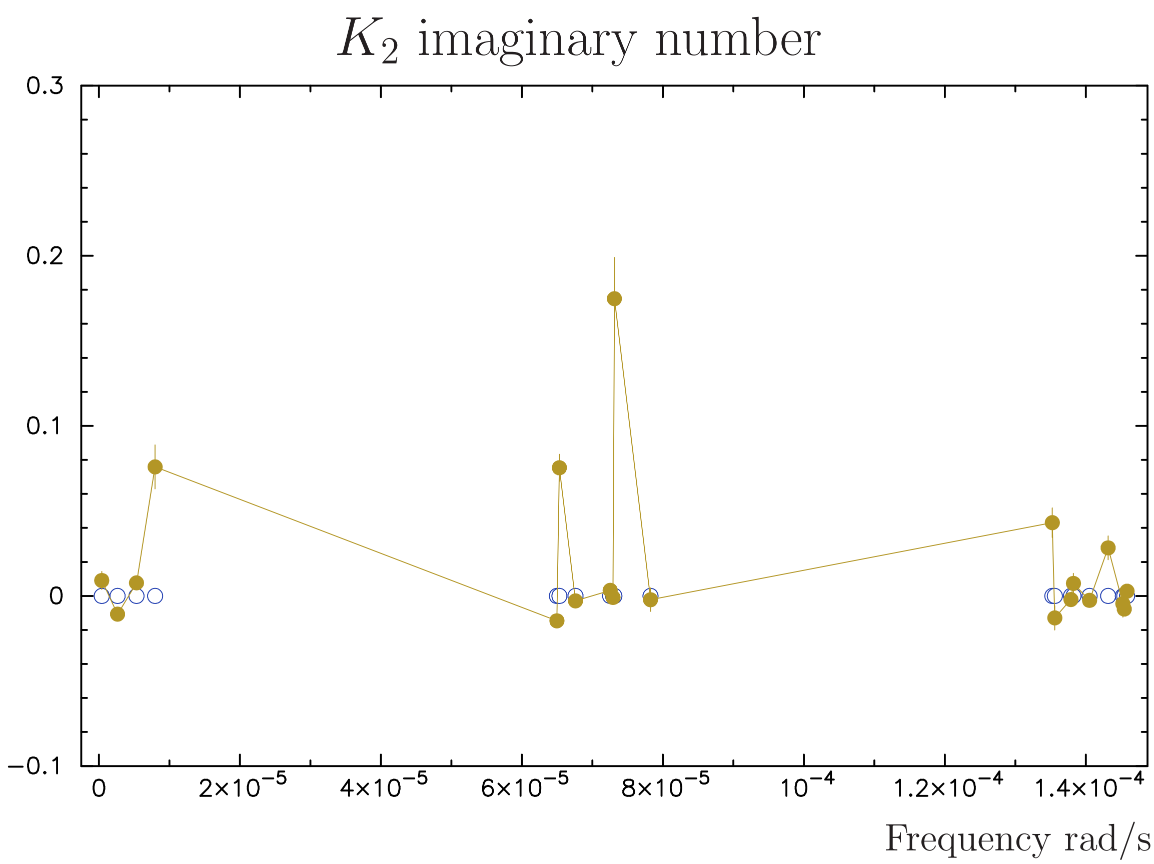



● — Gravimetry measurements

○ — Theory (DDW99in) predictions

# $K_2$ Love numbers from combination of VLBI and gravimetry




● — Results of observations analysis

○ — Theory (DDW99in) predictions

# Discrepancies with theories

- Semi-diurnal band, Love number $l_2$, real part:

| Tide | VLBI | DDW99 | Mathews2001 |
|---|---|---|---|
| $M_2$ | 0.0849 $\pm$ ( 0.0001 — 0.0003 ) | 0.0857 | 0.0847 |
| $S_2$ | 0.0847 $\pm$ ( 0.0002 — 0.0006 ) | 0.0857 | 0.0847 |
| $N_2$ | 0.0849 $\pm$ ( 0.0003 — 0.001 ) | 0.0857 | 0.0847 |
| $K_2$ | 0.0840 $\pm$ ( 0.0017 — 0.002 ) | 0.0857 | 0.0847 |
| All | **0.0848** $\pm$ ( 0.00007 — 0.0002 ) | 0.0857 | 0.0847 |

**Conclusion:** DDW99 deviates from observations at 99.9% significance level and **must be rejected**. Mathews2001 theory **has survived** experimental test.

- Semi-diurnal band, Love number $l_2$, imaginary part:

| Tide | VLBI | DDW99 | Mathews2001 |
|---|---|---|---|
| $M_2$ | -0.0004 $\pm$ ( 0.0001 — 0.0003 ) | 0.0 | -0.0007 |
| $S_2$ | 0.0008 $\pm$ ( 0.0002 — 0.0006 ) | 0.0 | -0.0007 |
| $N_2$ | -0.0002 $\pm$ ( 0.0003 — 0.001 ) | 0.0 | -0.0007 |
| $K_2$ | 0.0004 $\pm$ ( 0.0017 — 0.002 ) | 0.0 | -0.0007 |
| All | **-0.0001** $\pm$ ( 0.00007 — 0.0002 ) | 0.0 | -0.0007 |

**Conclusion:** Mathews2001 deviates from observations at 99% significance level and **must be rejected**. DDW99 theory **has survived** experimental test.

- Long period Love number $h_2$

| Tide | VLBI | Mathews2001 |
|---|---|---|
| Cw | 0.61 $\pm$ ( 0.02 — 0.04 ) | 0.625 |
| Ssa | 0.57 $\pm$ ( 0.02 — 0.13 ) | 0.621 |
| Mm | 0.622 $\pm$ ( 0.009 — 0.03 ) | 0.616 |
| Mf | 0.620 $\pm$ ( 0.005 — 0.03 ) | 0.613 |

**Conclusion:** Love numbers $h_2$ do not deviate from the theory at the 95% significance level.

- Long period Love number $l_2$

| Tide | VLBI | Mathews2001 |
|---|---|---|
| Cw | 0.098 $\pm$ ( 0.003 — 0.005 ) | 0.090 |
| Ssa | **0.104** $\pm$ ( 0.003 — **0.006** ) | **0.089** |
| Mm | 0.083 $\pm$ ( 0.002 — 0.004 ) | 0.087 |
| Mf | 0.091 $\pm$ ( 0.001 — 0.002 ) | 0.086 |

**Conclusion:** Love numbers $l_2$ for the semi-annual tide deviates from the theory at the 95% significance level.

# $K_1$ tide

- Diurnal band, $K_1$ tide, real part:

| Param | Observed value | DDW99 | Mathews2001 |
|---|---|---|---|
| $h_2$ | 0.526 $\pm$ ( 0.001 — 0.003 ) | 0.525 | 0.521 |
| $l_2$ | 0.0878 $\pm$ ( 0.0004 — 0.0006 ) | 0.0882 | 0.0873 |
| $\delta_2$–1 | 0.1365 $\pm$ ( 0.0003 — 0.002 ) | 0.1348 | 0.1349 |
| $k_2$ | 0.260 $\pm$ ( 0.0006 — 0.002 ) | 0.257 | 0.257 |

- Diurnal band, $K_1$ tide, imaginary part:

| Param | Observed value | DDW99 | Mathews2001 |
|---|---|---|---|
| $h_2$ | -0.001 $\pm$ ( 0.001 — 0.003 ) | 0.0 | -0.003 |
| $l_2$ | 0.0007 $\pm$ ( 0.0004 — 0.0006 ) | 0.0 | -0.0007 |
| $\delta_2$ | 0.0000 $\pm$ ( 0.0003 — 0.002 ) | 0.0 | 0.001 |
| $k_2$ | -0.001 $\pm$ ( 0.0006 — 0.002 ) | 0.0 | 0.001 |

Conclusion: no significant differences with theories at the level of 0.6%.

# Conclusions:

- For diurnal and semi-diurnal bands, VLBI gives the upper limit errors by a factor of 2–4 less than superconducting gravimetry measurements of the Earth's non-rigidity.

- Estimates of Love numbers at semi-diurnal band from VLBI allowed to discriminate theories of the solid Earth tides.

- Combination of results from VLBI and superconducting gravimetry allows to get estimates of Love numbers $k$ with the upper error limit for diurnal and semi-diurnal bands in the range 0.5–1.0%.