Approaches for modeling non-linear site position variations

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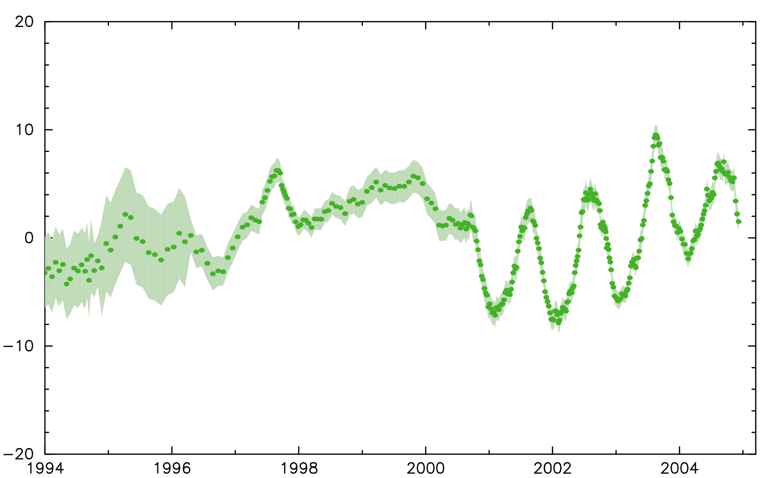
Abstract

Approaches for estimation of site motion beyond linear models are discussed. It is proposed to represent site motion in a sum of three functions: a linear function, an expansion with the B-spline bases, a sum of principal components of the expansion with the Fourier basis. The VLBI dataset from 1979 to 2005 was re-analyzed following this approach. It is concluded that this approach is adequate for describing the non-linear position evolution of VLBI stations.

1. Introduction

Traditionally, the site position evolution is described with a linear model. The technique for this type of motion estimation is well established. However, there is a number of pieces of evidence which show the inadequacy of this model is growing. There are certain phenomena which cause deviation of site motion from a simple linear model: 1) rail repair (for example, DSS65, MEDICINA which results in breaks in a position evolution; 2) seismic slips and transient deformations (TATEYAMA, GILCREEK); 3) seasonal changes in site positions due to mismodeling of mass loading; and 4) poorly understood local phenomena (for example, PIETOWN, HRAS_085). An example of non-linear site motion is shown in figure 1.

Figure 1. Evolution of the length of ALGOPARK/WESTFORD baseline after box-filtering with a width of 60 days. Units: mm.



Due to the presence of systematic errors the accuracy of determination of linear velocity for many stations is approaching the correlation limit. Therefore, no significant accuracy improvement in the framework of the linear model is expected. On the other hand, there is a demand from the geophysical community to understand better seasonal and inter-annual station position variations. This motivates us to seek an alternative method for site positions estimation.

2. Analysis of the problem

It may seem sufficient to estimate a daily time series of VLBI site positions and then analyze it. However, we should be aware of several complications. The equations of light propagation are differential, they do not determine the solution, but rather a family of solutions. In order to select a member in this family, boundary conditions should be imposed, either explicitly or implicitly in the form to fixing six linear combinations of site coordinates to their apriori values. These boundary conditions make the estimates of site position linearly dependent. This means that variations of position of site A will cause apparent variations in *estimates* of position of site B. These induced variations in estimates of other sites depend on the network configuration and the number of stations. The induced variations in estimates of coordinate of site B due to variations in position of site A will be diluted approximately as $\frac{1}{n}$, where n is the total number of sites in the network. Each network of VLBI sites which participate in an individual experiment is small, typically 5–8 stations, so induced variations are nor diluted enough to become negligible. To make things worse, the VLBI network is constantly changing: the set of stations which participate in experiments varies from day to day. These factors make interpretation of changes in *coordinates* as a physical *displacement* very unreliable.

Another complication arises when the Earth orientation parameters are evaluated. The small vector of the Earth rotation determined by the pole coordinates and UT1 is equivalent to the net-rotation of the network of observing stations with respect to some apriori positions. It should be stressed that removal or addition of a station changes the net-rotation of the network which is of the order of magnitude $\frac{\Delta r}{n}$ where Δr is the adjustment to position of an additional site, and n — the total number of stations in the network. Since typically the VLBI network configuration is not held the same in different experiments (during 1979–2005 1240 network configurations were used in 3900 experiments), the Earth orientations parameters evaluated at experiments in different network configurations will have jumps which depend on apriori site coordinates. This will make their interpretation problematic.

Finally, allowing site position to evolve freely on a daily basis reduces the strength of the solution, since it its dimension increases. But there is no reason to believe that stations move chaoticly at the time scale of 1–30 days. The estimation strategy which evaluates site position independently at adjacent epochs implicitly assumes that stations may have arbitrary large displacements between epochs, and therefore, have arbitrary large accelerations caused by arbitrary large forces. Analysis of the physics of motion can help us to come out with a more realistic model.

2.1. What can cause non-linear site motion?

Understanding causes of non-linear motion helps us to find the optimal parameterization of such a motion. Known causes of the motion can be characterized the following way:

- Antenna displacements due to human activity. Antenna repair can cause an abrupt shift in antenna position at a known epoch.
- Seismic motion. Such a motion is characterized as a sudden jerk in station position at precisely a known epoch which may be followed by an exponential relaxation at time scales

from months to several years.

- Loading mismodeling. Mismodeling an ocean loading results in harmonic site position variations with precisely known frequencies. As was shown in [2], residual errors of ocean loading models are within 1–3 mm. Mismodeling atmospheric pressure loading results mainly in a quasi-random motion with amplitudes with rms less than 1 mm according to [3]. Mismodeling a hydrology loading may result in annual, semi-annual and in some cases ter-annual position variations with an amplitude of 2-8 mm. Currently, it is rather difficult to evaluate the accuracy of hydrological models.
- Unaccounted thermal deformations of the antenna. This signal is mainly diurnal and annual.

3. Mathematical model for non-linear motion

The preferable mathematical model for a non-linear motion should utilize the apriori knowledge about the process under consideration. The solution is sought in the form of an expansion the process with an *appropriate* basis. The coefficients of the expansion are evaluated by using least squares together with linear site velocities, positions at the reference epoch, source coordinates, Earth orientation, and nuisance parameters which characterize clock function and atmospheric path delay. A poorly selected basis requires a dimension of the expansion comparable with the dimension of the initial problem. The process expanded with the appropriate basis will have non-principal components vanishing which allows us to reduce significantly the dimension of the problem.

In this paper I propose to use two basises: the Fourier basis and the B-spline basis.

The components of the Fourier basis which corresponds to the tidal, annual and diurnal frequencies and their multiples are included in the parametric model. These constituents represent the residual tidal, seasonal and diurnal signals. The Fourier basis is the eigenfunction of the linear operator, and it is suitable for describing phenomena which are incessant in the range $[-\infty, +\infty]$.

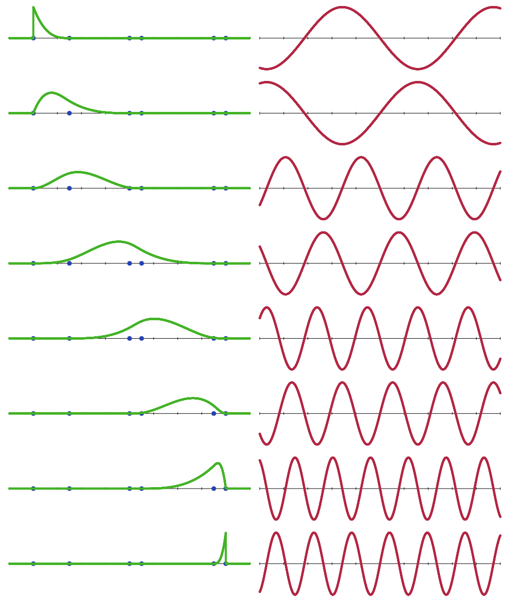
The B-spline basis is suitable for modeling events localized in time. We know [1] that 1) a B-spline of degree k is non-zero at $k+1$ knots; 2) a B-spline of degree $2k-1$ provides a minimum to the functional $\int\limits_a^b [f(x)^{(k)}]^2 dx$ which takes specific values at the sequence of knots. In that sense the B-spline is the most smooth function used for approximation.

Since we simultaneously solve for site positions, linear velocities and coefficients of B-spline, we have to impose decorrelation constraints requiring the zeroth and first momentum over the basis functions to be a constant:

$$\int\limits_a^b \sum_i B_i(x)\, dx = \text{constant} \tag{1}$$

$$\int\limits_a^b \sum_i x\, B_i(x)\, dx = \text{constant} \tag{2}$$

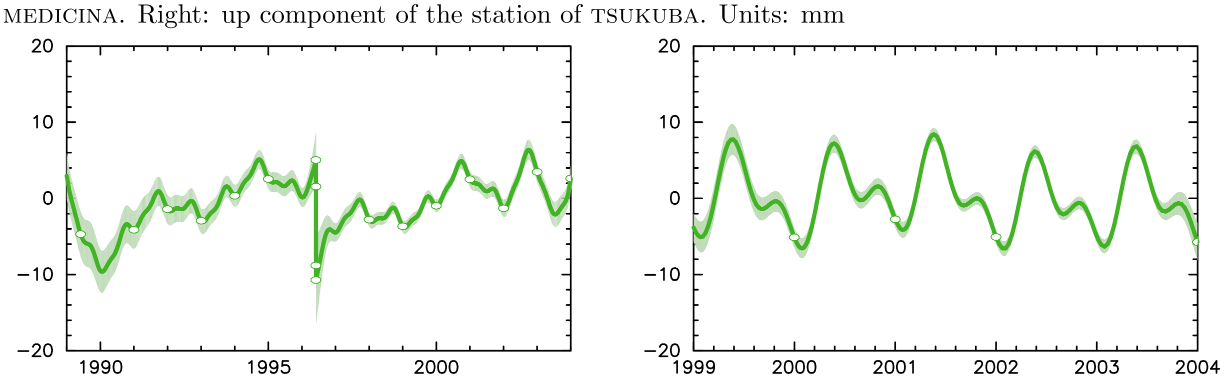
Figure 2. Example of basis functions: the B-spline at six nodes (left) and eight Fourier basis functions



4. Result of analysis

The ability to model non-linear site position variations was added to the VLBI analysis software Calc/Solve[1] in 2005. In trial solutions all VLBI observations from 1979 to 2005 were used. In addition to the traditional set of parameters (site positions at reference epoch, site velocities, the Earth orientation parameters, clock function, atmosphere zenith path delay and the atmosphere tilt), sine and cosine amplitudes at annual, semi-annual, diurnal, semi-diurnal frequencies were evaluated as well as the coefficients of the B-spline expansion of the third degree for 36 sites. The knots of the B-spline were selected equidistantly with the time span of 2 years, except the stations with seismic events, for example, GILCREEK, and stations with rail repairing, DSS65, MEDICINA. For these stations the knots with the multiplicity three were inserted at epochs of events.

Figure 3. The empirical model of non-linear changes in site coordinates. Left: east component of station MEDICINA. Right: up component of the station of TSUKUBA. Units: mm

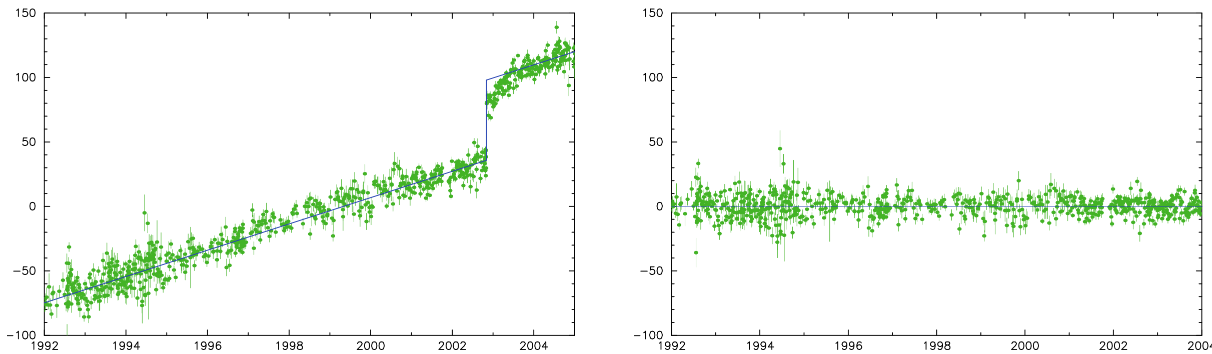


[1] http://gemini.gsfc.nasa.gov/solve

Examples of the estimates of the empirical model of the non-linear motion are shown in figure 3. Station MEDICINA has a complicated motion: a break caused by rail repairing on 1996.06.01, seasonal variations and inter-annual variations. Non-linear motion of the station of TSUKUBA is mainly seasonal which maybe related to the seasonal changes of the ground water level related to rice fields irrigation.

The empirical model of station motion was substituted in a separate baseline solution. Figure 4 shows results of the substitution for the baseline GILCREEK/WETTZELL affected by Denali earthquake on 2002.11.03.

Figure 4. Evolution of the baseline GILCREEK/WETTZELL. Left: the baseline length time series without modeling non-linear motion. Right: the estimates of baseline lengths after applying the empirical model. Units: mm



5. Conclusions

It was found that the technique of estimation of the coefficients of expansion with the B-spline and Fourier basis is adequate for modeling non-linear site position variations. An addition of new parameters causes a moderate increase of dimension of the problem, but retrieves the signal. When the time span for B-spline is selected to two or even one year, the solution is still stable. 40–50 stations participate in observation during one calendar year. This estimation technique is equivalent to analysis of observations for 1–2 years at a 40–50 station network, in contrast to 4–7 station networks used in each daily individual experiment.

Several problems are still to be solved theoretically: 1) what is the optimal choice of epochs for knots of the B-spline basis?; 2) should constraints on time derivatives of B-spline be applied?; 3) how many harmonics can be included in a solution before it becomes instable?

References

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