Using a Numerical Weather Model to Improve Geodesy

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Abstract. The use of a Numerical Weather Model (NWM) to provide in situ atmosphere information for mapping functions of atmosphere delay has been evaluated using Very Long Baseline Interferometry (VLBI) data spanning eleven years. Parameters required by the IMF mapping functions (Niell 2000, 2001) have been calculated from the NWM of the National Center for Environmental Protection (NCEP) and incorporated in the CALC/SOLVE VLBI data analysis program. Compared with the use of the NMF mapping functions (Niell 1996) the application of IMF for global solutions demonstrates that the hydrostatic mapping function, IMFh, provides both significant improvement in baseline length repeatability and noticeable reduction in the amplitude of the residual harmonic site position variations at semidiurnal to long-period bands. For baseline length repeatability the reduction in the observed mean square deviations achieves 80% of the maximum that is expected for the change from NMF to IMF. On the other hand, the use of the wet mapping function, IMFw, as implemented using the NCEP data, results in a slight degradation of baseline length repeatability, probably due to the large grid spacing of the NWM that is used.

1 Introduction

The accuracy of the estimate of the local vertical coordinate from GPS and VLBI observations increases as the minimum elevation cutoff of included data is reduced, due to improved decorrelation of the estimates of troposphere path delay and the site vertical coordinate. From the other side, the errors in mapping the zenith troposphere delay increase rapidly with decreasing elevation angle and cause a degradation of the precision of the estimate of site position coordinates. Thus, it is important to assess the optimum minimum elevation above which data should be included.

The total atmosphere path delay is expressed as the product of the zenith path delay and the mapping function, $m(e)$, defined as the ratio of the excess path delay at geometric elevation, e, i.e., the vacuum elevation, to the path delay in the zenith direction. The hydrostatic and wet mapping functions are calculated separately because the vertical distributions of refractivity differ (MacMillan and Ma, 1994; Niell, 1996). Many geodetic analyses for GPS and VLBI currently use very simple models for mapping functions that are based on site location and time of year (Niell, 1996; hereafter referred to as NMF). This climatological approach is used due to the difficulty of obtaining in situ meteorological information along the path traversed by the incoming radio waves.

The next best thing would be mapping functions based on the vertical distribution of refractivity above the site, assuming azimuthal symmetry. While measurement of such a profile, for example by launching radiosondes several times per day, has not been deemed feasible, some amount of information on the state of the atmosphere can be obtained from the meteorologic numerical weather models (NWM), which provide estimates of the vertical distribution of temperature, pressure, and water vapor content at specified horizontal grid points over the entire globe every six hours. Niell (2000, 2001) demonstrated how this information can be used to improve the mapping functions.

In this note we show that these mapping functions (IMF), using data from NCEP on a $2.^\circ5$ by $2.^\circ5$ grid interpolated to the site location, provide the expected improvement in baseline length repeatability, and that, furthermore, they provide a noticeable reduction in the amplitudes of residual harmonic site position variations.

While the wet mapping function (IMFw) has the potential to be an improvement over NMFw, as demonstrated using radiosonde profiles (Niell, 2000), this is not achieved in the current implementation using the NCEP NWM in the VLBI analysis package CALC/SOLVE (Ma et al., 1990).

In section 2 we give a brief description of IMF and in section 3 outline the implementation in CALC/SOLVE. The VLBI analysis is described in section 4 and some of the geodetic implications are presented in sections 5. Section 6 contains a summary of the results and suggestions for future developments.

2 The Isobaric Mapping Functions (IMF)

Currently the closest approximation to a "true" mapping function is obtained by using radiosonde data to define the vertical profile of radio refractivity and, assuming spherical symmetry about the radiosonde launch site, integrating along the path determined by the refractivity to determine the delay through the atmosphere both in the zenith direction and at the elevation for which the mapping function is to be calculated. The delays due to the hydrostatic and wet components of the atmosphere (Niell 1996) are retained independently in the raytrace integration, and the mapping functions are calculated separately.

Marini (1972) showed that, for a spherically symmetric but vertically arbitrary profile of atmospheric refractivity, the mapping function can be approximated by a continued fraction of the form

$$m(e) = \frac{1 + \cfrac{a}{1 + \cfrac{b}{1 + c}}}{\sin e + \cfrac{a}{\sin e + \cfrac{b}{\sin e + c}}} \qquad (1)$$

The numerator is included to normalize the fraction at an elevation angle of 90°, and the number of terms (only three are shown) should be determined by the desired accuracy of the fit. Niell (1996) found that three terms is sufficient to keep the error to less than 1 mm for elevations down to 3° if the delays are calculated at eight elevations in addition to the zenith direction.

The goal of mapping function research is to find the functional form and dependence on external information of the parameters a, b, c ... that best matches the actual values of the mapping function for real atmospheres at arbitrary locations and times.

In order to provide a useful tool for estimating or calculating the atmosphere path delay for geodetic observations, the parameters a, b, and c should be calculable in terms of available information. The following procedure was used to determine the relation of the parameters to external data.

1. For radiosonde profiles from a large number of sites and spanning at least one year, calculate, by raytracing, the hydrostatic and wet mapping functions at nine elevations from 90 degrees down to 3 degrees.

2. Calculate by least-squares fit the continued fraction parameters a, b, and c for each profile for both hydrostatic and wet mapping functions.

3. Assume that the parameters are linear functions of some other parameters, q_i, that are available to characterize the mapping function, such as geographic location, time of year, or a combination of meteorological parameters. Expand each of the parameters a, b, and c in terms of the q_i.

4. Use linear regression of all a values on parameters q_i for all sites and profiles to determine the regression coefficients that describe a in terms of the q_i. Repeat for b and c; do for both the hydrostatic and wet parameters.

To determine the parameter dependence, the hydrostatic and wet mapping functions were calculated from radiosonde profiles for twenty-six sites for a one year period (1992) with usually two profiles per day, at 00 and 12 UT. Based on the expectation that the hydrostatic mapping function would be strongly correlated with the thickness of the atmosphere, a possible empirical relation to the geopotential heights of constant pressure levels (isobaric surfaces) for each of the parameters a_h, b_h, and c_h was investigated. The 200 hPa level gave the best agreement (Niell, 2000). The relation was found to require a dependence on the cosine of twice the latitude. The coefficients relating the parameters

a_h, b_h, and c_h to the geopotential height and the latitude were obtained by linear regression on the 200 hPa heights from the radiosonde data and *cos(2*latitude)*. Since the 200 hPa isobar has little sensitivity to the topography below, a correction for height of a site above sea level was calculated by linear regression of the site heights and the residual mapping function error using the derived coefficients. This was found to be consistent with that used for NMF, and the NMF height correction (Niell 1996) was adopted.

Although the concept of atmospheric thickness failed to produce a reasonable relation for the wet mapping function, we found that there exists a linear regression between the parameters a_w, b_w, and c_w and a "wet pseudo-mapping function". The wet pseudo-mapping function, ρ, is given by

$$\rho = \frac{r(3.^{\circ}3)}{r(90^{\circ})} \tag{2}$$

where

$$r(\epsilon) = \frac{k_3 \int \frac{e_v(h)}{T^2(h)} \frac{1 + \frac{h}{R_\oplus}}{\sqrt{\left(1 + \frac{h}{R_\oplus}\right)^2 - \cos^2 \epsilon}} \, dh}{k_3 \int \frac{e_v(h)}{T^2(h)} \, dh} \tag{3}$$

ρ is the ratio of the integral of the wet refractivity along an elevation of 3.3° to the integral in the vertical direction, and $r(\epsilon)$ is the integral of wet refractivity along elevation angle ϵ. Here e_v is specific humidity, T is temperature, h is height, and the integration is done through the whole atmosphere. $r(\epsilon)$ is a pseudo-mapping function, because bending of the raypath due to the spatial change in the index of refraction is not included in the calculation. This parameter was investigated in order to be able to avoid the large amount of computing required for an accurate ray-trace.

These regression coefficients allows us to compute mapping functions using geopotential height and the ratio of the wet refractivity integrals obtained from a numerical weather model (NWM) for the case when no radiosonde atmosphere profile near an observing station is available, as is usually the case during VLBI and GPS observations.

3 Implementation at GSFC for VLBI

The NWM data used are the gridded data from the U.S. National Center for Environmental Protection (NCEP) that are output every six hours. We used the Reanalysis model (Kalnay et al., 1996) in our study. It appears on their Web site with a time lag of 3 to 7 days. All Reanalysis model products have been analyzed with a consistent, though not necessarily the most recent, model. The current model was adopted in 1996. The files that are downloaded contain geopotential height and temperature at eighteen pressure levels, specific humidity at eight pressure levels, and the surface height for each grid point. The values of geopotential height at 200 hPa are extracted, and the values of ρ are calculated at each grid point. For the VLBI application the parameters are subsequently interpolated to the horizontal position of each VLBI site at each NCEP epoch (0, 6, 12, 18 UT). During interpolation, the unit vector of the normal to the surface of geopotential height at 200 hPa is computed. We assume that the hydrostatic atmosphere path delay is azimuthally symmetric with respect to the normal to the geopotential height, rather than local zenith. Thus, we imply that the atmosphere is tilted with respect a local observer. Typical values of the tilt are 0.1 to 1.0 mrad.

4 VLBI analysis and results

The troposphere path delay at each station is modeled as a linear spline with typical time span of 20 minutes, and parameters of the linear spline are estimated together with other parameters of interest in a single least-squares solution. The hydrostatic mapping function is used for computing the apriori hydrostatic path delay (the apriori wet path delay is set to zero) and the wet mapping function is used as the partial derivative for estimation of the wet zenith path delay. If the hydrostatic mapping function is wrong, then the apriori total path delay in the direction of the observed source will be biased. Since the hydrostatic mapping function and the wet mapping function are slightly different at low elevations (the difference is 4–7% at elevation 5°), the estimation procedure will not remove this bias completely. Thus, systematic errors in the hydrostatic mapping function result in systematic errors in estimates of site position. Due to the errors in the wet mapping function, which is used as the partial derivative, the contribution of the wet troposphere path delay to the observables is not completely removed by estimating the wet path delay in the zenith direction.

Figures 1 and 2 show the differences between the hydrostatic NMF and IMF mapping functions at two sites. If the IMF is closer to the true mapping function, we could expect that using IMF would reduce systematic errors in GPS and VLBI results.

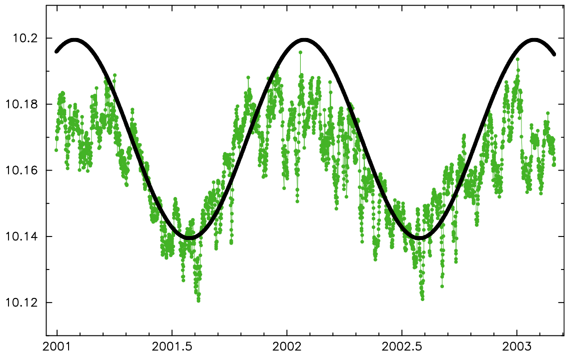

Figure 1: NMFh (thick line) and IMFh (dots) at elevation angle 5° at station Gilcreek (Alaska).

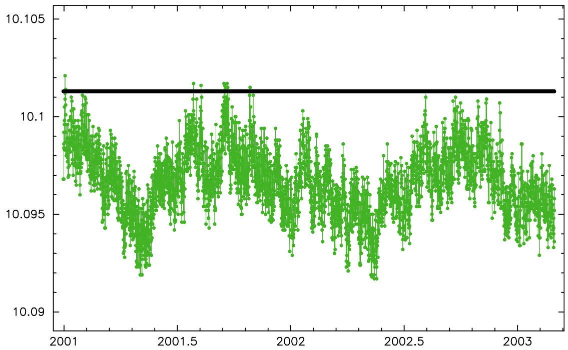

Figure 2: NMFh (thick line) and IMFh (dots) at elevation angle 5° at station Fortleza (Atlantic coast, Brazil).

To evaluate *quantitatively* what kind of improvement in geodetic results is provided by the use of IMF instead of NMF, we performed a series of VLBI solutions and investigated changes in baseline length repeatability and changes in the estimated amplitudes of the harmonic site position variations.

4.1 Baseline length repeatability test

First, we need to evaluate how a replacement of the NMFh with IMFh affects baseline lengths. For this purpose we made a special solution. We used the standard parameterization in the estimation model: we solved for site positions, troposphere path delay modeled by spline of the first degree with time span 20 minutes, clocks, Earth orientation parameters, source positions, and other nuisance parameters. The hydrostatic zenith path delay was computed using the formula of Saastamoinen (1972), and IMFh was used to transform the apriori zenith hydrostatic path delay to the path delay in the source direction. However, in the right hand-side of the equations of conditions, instead of the differences between observed and modeled delay we inserted the differences between hydrostatic path delays calculated using NMFh and IMFh but with the same apriori zenith hydrostatic path delay computed from the Saastamoinen formula. All VLBI observations for the period of 1993–2003 were used. We obtained from this solution the series of baseline length estimates which have the meaning of changes in baseline lengths due to changes in the hydrostatic mapping function, and we computed the weighted mean value of the length change for each baseline. Since replacement of the mapping function affects mainly the estimate of vertical site coordinate, the changes in baseline length, δl, will depend on baseline length as $\Delta l = \dfrac{L}{2R_\oplus}\,(\Delta h_1 + \Delta h_2)$, where L is the baseline length and $R_\oplus$ is the radius of the Earth. Figure 3 shows the mean changes in baseline length as a function of the total baseline length for all baselines in the solution.

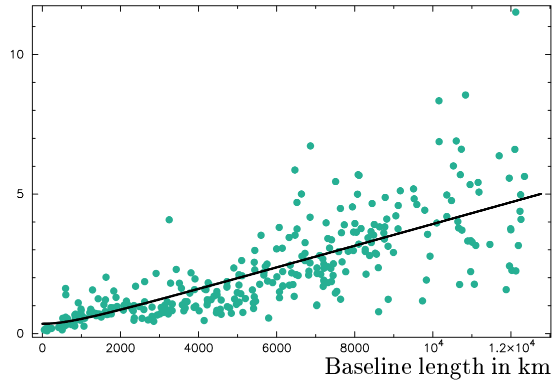

Figure 3: Changes in baseline lengths due to replacement of NMFh with IMFh (in mm) as a function of baseline length.

We then made two other solutions. In solution A we used NMFh and in solution B we used IMFh. Parameterization was exactly the same as in the previous special solution. For each baseline we got the series of length estimates and evaluated the formal uncertainties of these estimates. A linear model was fitted to each series, and the wrms deviation from the linear model was computed for each baseline. We call this wrms baseline length repeatability. We formed the differences of the squares of repeatabilities in the sense solution A (IMFh) minus solution B (NMFh). The differences, smoothed with a Gaussian filter, are presented as a function of baseline length in figure 4.

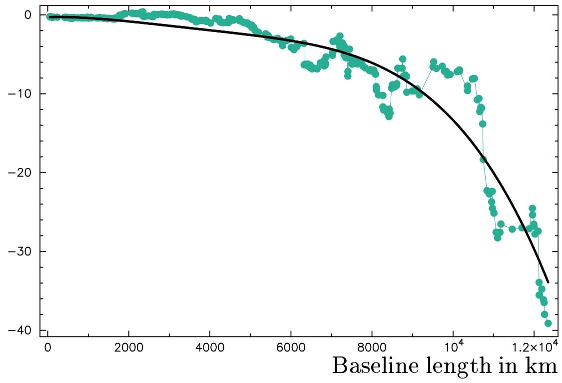

Figure 4: Differences in squares of baseline lengths in mm^2 using IMFh versus NMFh for computing apriori hydrostatic path delay, in the sense IMFh-NMFh.

We see that the differences are negative. This means that using IMFh instead of NMFh reduced the baseline length repeatability and, thus, improved the quality of the geodetic results. But we would like to have a *quantitative measure* of how replacement of NMFh by IMFh brings us closer to the true mapping function. We introduce a coefficient of reduction of variance R (see Petrov and Boy (2003) for more details) as a quantitative measure of changes in baseline length repeatability:

$$R = \frac{\Delta\sigma^2 + \sigma_m^2}{2\,\sigma_m^2} \qquad (4)$$

where $\Delta\sigma^2$ is the difference in the squares of baseline length repeatability of the solution with NMFh minus the solution with IMFh, and σ_m^2 is the square of differences in baseline lengths, derived from the special solution, which has the meaning of the square of expected changes in baseline lengths. This coefficient runs from 0 to 1. In the case that the change of mapping function did not result in a change of baseline length repeatability, the coefficient is 0.5. In the case that NMFh is much closer to the (unknown to us) "true" mapping function than IMFh is, the use of IMFh will result in degradation of baseline length repeatability by the amount σ_m^2, and then $R \approx 0$. If IMFh is closer to the true mapping function than NMFh by an amount equal to the difference (NMFh - IMFh), then the difference in baseline length repeatability is equal to σ_m^2, and $R = 1$.

We have computed the coefficient of reduction of variance for each baseline observed 16 or more times and have calculated the weighted mean value of R over all baselines. We found that $R = 0.81 \pm 0.02$. This means that 80% of the power in the change of baseline lengths due to the change in mapping function is present in the data, and the baseline length repeatability has improved by approximately 80% of the expected change under the assumption that the new mapping function, IMFh, is closer to the true mapping function than the old NMFh by the amount of the differences between these two functions.

This result is very encouraging. It demonstrates that using only one *value*, the geopotential height of the 200 hPa surface, we can reconstruct a mapping *function* (an infinite set of values) with a surprisingly good precision: not worse than 80%.

4.2 Changes in harmonic site position variations

As we see in figures 1 and 2, NMFh does not model seasonal changes perfectly, and it completely ignores harmonic variations of the mapping function at frequencies others than annual. Thus, we can expect that these errors will cause harmonic variations in the estimates of site positions. As we saw in the previous section, the use of IMFh improves baseline length repeatability. Therefore we can expect that observed residual harmonic site position variations will be reduced when IMFh is used instead of NMFh.

In order to evaluate this effect quantitatively, we made two solutions. In both solutions we estimated troposphere path delay, clock parameters, Earth orientation parameters, station position and velocities, source coordinates, and harmonic site position variations at 32 frequencies for each station. NMFh/NMFw were used in the first solution, and IMFh/NMFw were used in the second solution. We included all observations from 1980 through 2003 for 40 VLBI stations with a long history of observations. Our theoretical model included site displacements due to solid Earth tides, ocean loading, atmosphere loading, and hydrology loading. Thus, we obtained residual harmonic site position variations.

For each frequency we computed the ratio of the weighted sum of squares of the observed amplitudes of harmonic site position variations to its mathematical expectation, χ^2/f. In the absence of a signal χ^2/f should be less than 1.2. Detailed analysis of the technique for solutions of this type can be found in Petrov and Ma (2003).

We noticed differences in these statistics at six frequencies. They are presented in table 1. We see that the power of the residual signal is reduced at all frequencies when IMFh is used instead of NMFh. To our surprise the power was reduced not only at annual (Sa), diurnal (S_1) and semi-diurnal (S_2) frequencies, where we expected improvement, but also at semi-annual (Ssa), sidereal (K_1) and semi-sidereal (K_2) frequencies. Changes of the power of residual signal at the K_1 frequency may have a significant consequence: it means that using IMFh instead of NMFh may result in a non-negligible change in the estimate of the precession rate, since precession corresponds to the harmonic variation in the Earth rotation at the sidereal frequency.

Table 1: χ^2/f statistics of residual harmonic site position variations in two solutions which used different mapping functions.

Tide	χ^2/f			
	NMFh, NMFw		IMFh, NMFw	
	vert	horiz	vert	horiz
K_2	1.80	2.01	1.61	2.05
S_2	2.55	1.73	2.15	1.76
K_1	2.13	3.45	1.89	3.34
S_1	3.80	2.32	3.58	2.22
Ssa	2.42	1.12	1.85	1.09
Sa	6.34	2.56	5.77	2.43

4.3 Changes in the scale factor of the TRF

Systematic changes in the estimates of the vertical component of site positions may result in a change of the scale factor of the output site position catalogue. In order to evaluate quantitatively this effect, we have analyzed the catalogue of site positions obtained in the solutions described in the previous section. We estimated the 7-parameter transformation of the site position catalogue from the solution that used IMFh with respect to the catalogue from the solution that used NMFh. This transformation includes translation, rotation, and the scale factor. We found that for the site position catalogue from solutions with IMFh the scale is greater by 0.21 ± 0.05 ppb with respect to the solutions using NMFh. This changes in the scale factor corresponds to an approximately 1 mm increase of vertical coordinates of site positions.

4.4 Wet IMF

We made the baseline length test for using IMFw in place of NMFw, but instead of improvement in the baseline length repeatability, we found a slight degradation. It should be noted that the expected improvement, 1.0 mm at baseline lengths 10000 km, is very small, a factor of 5 less than the improvement due to replacement of the hydrostatic mapping function. At the same time comparison of IMFw and NMFw with respect to the mapping function derived using radiosonde data at the sites of radiosonde launches indicates that IMFw better approximates the radiosonde mapping function than NMFw, in contradiction to the results of the VLBI data analysis. We think that this contradiction is related to the fact that the relatively large grid of the NCEP Reanalysis NWM, 200x270 km at mid-latitudes, is not adequate to represent the wet refractivity field.

5 Geodetic implications

If there were no model errors, it would be advantageous to make observations to as low an elevation as permitted by the equipment and the physical surrounding, for both GPS and VLBI. Including the widest range of elevations reduces the uncertainty in the height estimate and improves separation of the estimates of the vertical component of position from the estimates of the zenith path delays and the clock offsets. However, any error in the azimuthally symmetric atmosphere model will propagate into errors in the estimated vertical coordinate, and these errors will increase with decreasing minimum elevation. Thus, there is a trade-off between atmosphere uncertainty and the accuracy of determination of the vertical, and it is possible to choose the optimum minimum elevation if the error model is sufficiently well known.

The formal uncertainty in the vertical, taking into account atmosphere stochastic variability, assuming the clock is modeled as white noise, and adding a fixed amount of delay noise, is calculated in the precise point positioning mode (PPP) of the GPS analysis software Gipsy (Zumberge et al., 1997), although other effects, such as satellite orbit error, are not included explicitly. For twenty-four hour sessions the uncertainties for minimum elevations of 15° down to 5° decrease from 4.0 mm to 2.0 mm (the data were taken with an Ashtech Z-12 with good sky visibility).

The effect of the atmosphere model error has been calculated by comparing the IMF and NMF mapping functions obtained from the NWM at twenty-six radiosonde launch sites for the year 1992, with the mapping functions calculated by raytracing along the path of the radio waves at various elevations, assuming a vertical distribution of refractivity given by the radiosonde temperature, pressure, and humidity profiles. Using the surface pressure for the zenith hydrostatic delay, and the zenith wet delay from the radiosonde data, the rms delay errors for an elevation of 5° are shown in figure 5 for the hydrostatic component of IMF and for the hydrostatic and wet components of NMF. The errors for IMFw are very close to those of NMFw.

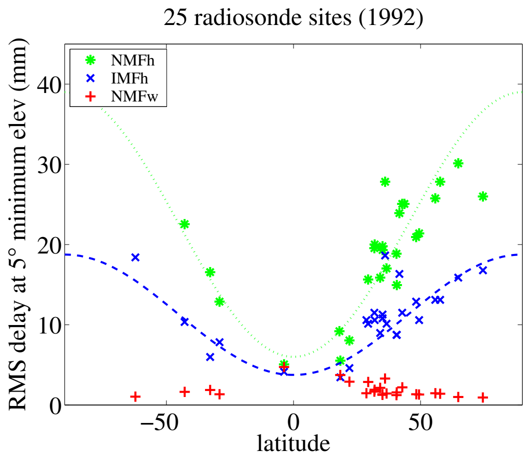

Figure 5: Rms delay error for mapping functions at radiosonde sites for 5° elevation.

How do these delay errors affect the height estimates? For errors that increase with decreasing elevation, such as those that are due to the atmosphere, the lowest elevation observations have the greatest effect on the height uncertainty. As a result, the effect on the height uncertainty can be calculated as the product of the delay error at the lowest observed elevation and the sensitivity of the height estimate to delay change at that elevation.

MacMillan and Ma (1994) evaluated the sensitivity of the vertical error to the error in atmosphere path delay at the minimum elevation. They found that an error of 1 mm in atmosphere path delay at the minimum elevation angle causes 0.22 mm error in the vertical coordinate if the minimum elevation is 5°, and 0.4 mm if the minimum elevation is 10°. We made a similar test for a later set of VLBI data and found a sensitivity of 0.3–0.35 for a minimum elevation of 5°. A simulation of GPS observations using a simple model of estimating vertical, zenith troposphere delay, and clock for a single epoch gave sensitivity of 0.4 to 0.8 over the minimum elevation range 5°–15°. For GPS the actual sensitivity will depend on what satellites are visible at the time of observation, but for illustration of the technique we will use the values from our GPS simulation.

To show the effect of the mapping function errors on height at mid-latitude the rms delay differences were found between both the NMF and IMF mapping functions and mapping functions obtained by raytracing the radiosonde data (as described in section 2) for the site ALB (Albany, state New York, USA) at latitude 42°. The differences were converted to height uncertainties using the sensitivities described in the previous paragraph (figure 6). We calculated inflated formal uncertainties of the height estimates by adding in quadrature the point-positioning uncertainties and the uncertainty due to errors in the mapping function. These total errors are also shown in figure 6. From this study we conclude that the use of NMFh significantly increases the total height uncertainty when data are included below about 10° for a twenty-four hour session. At the same time the use of IMFh does not result in an increase of total height uncertainties until data below about 7° are included.

This type of evaluation can be important for deciding how much data to include in a solution. For example, the lower elevation data might be desirable for improving estimates of atmosphere gradients, but only if the uncertainty of vertical estimates is not degraded.

An important point to notice from figure 6 is that for a minimum elevation of 15°, which is the standard for many GPS analyses, the error in the

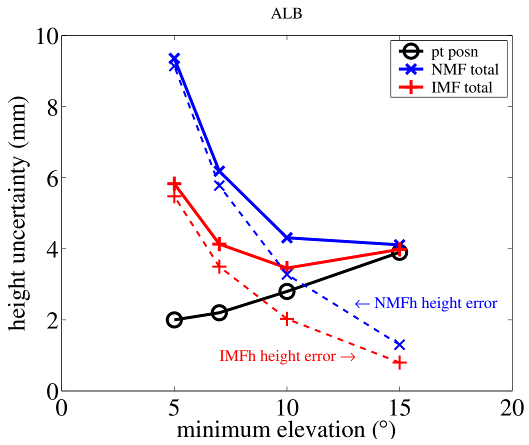

Figure 6: Height uncertainty for different minimum elevations for precise point-position and for mapping functions.

vertical estimation due to the atmosphere is less than 2 mm and is therefore not significant for daily solutions. However, in computing weekly site position averages, the atmosphere modeling error may contribute more than expected since weather systems often persist over many days, and the actual uncertainty of the average will decrease more slowly than $\frac{1}{\sqrt{n}}$, because the errors are correlated over longer than one day.

6 Summary, conclusions, future work

Atmosphere delay mapping functions, designated IMF, based on in situ meteorological information from the NCEP numerical weather model, have been implemented in the VLBI analysis package CALC/SOLVE. When applied to global VLBI solutions that include all data above a minimum elevation of 5°, their use, compared to the model NMF, improves baseline length repeatability and reduces the power in residual harmonic site position variations.

If used for GPS, IMF will allow lower elevation data (down to approximately 7°) to be included in the solutions while maintaining the formal uncertainty in the vertical that is obtained for 15° minimum elevation, and will reduce amplitudes of observed residual harmonic site position variations at annual and semi-annual periods.

With improvement in modeling of annual and semi-annual atmosphere errors, it is time to begin modeling systematic temperature-dependent antenna height errors for GPS and VLBI.

7 Acknowledgment

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