Determination of Love numbers H and L for long-period tides using VLBI

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- **Problem:**

  VLBI measures station displacements with high precision. Analysis of all available observations allows detection of even weak tidal signals.

  The purpose of this study is to determine from observations Love numbers h_2 and ℓ_2 for

  - zonal tides with periods from 9 days to 18.6 years;
  - pole tide.

  Motivation:

  - to answer the question: whether Love numbers for long periods are as large as theory predicts;
  - to assess the long-term stability of VLBI.

- ## Observations:

  Dense series of all available dual-band Mark-III/Mark-IV VLBI observations for 16.6 years (1984.0–2000.6) plus sparse observations (1979.8–1983.9) were used in analysis.

  About 3000 24-hour experiments with 3 million measurements of time delay were used in analysis.

Global VLBI network

- # Model of reduction:

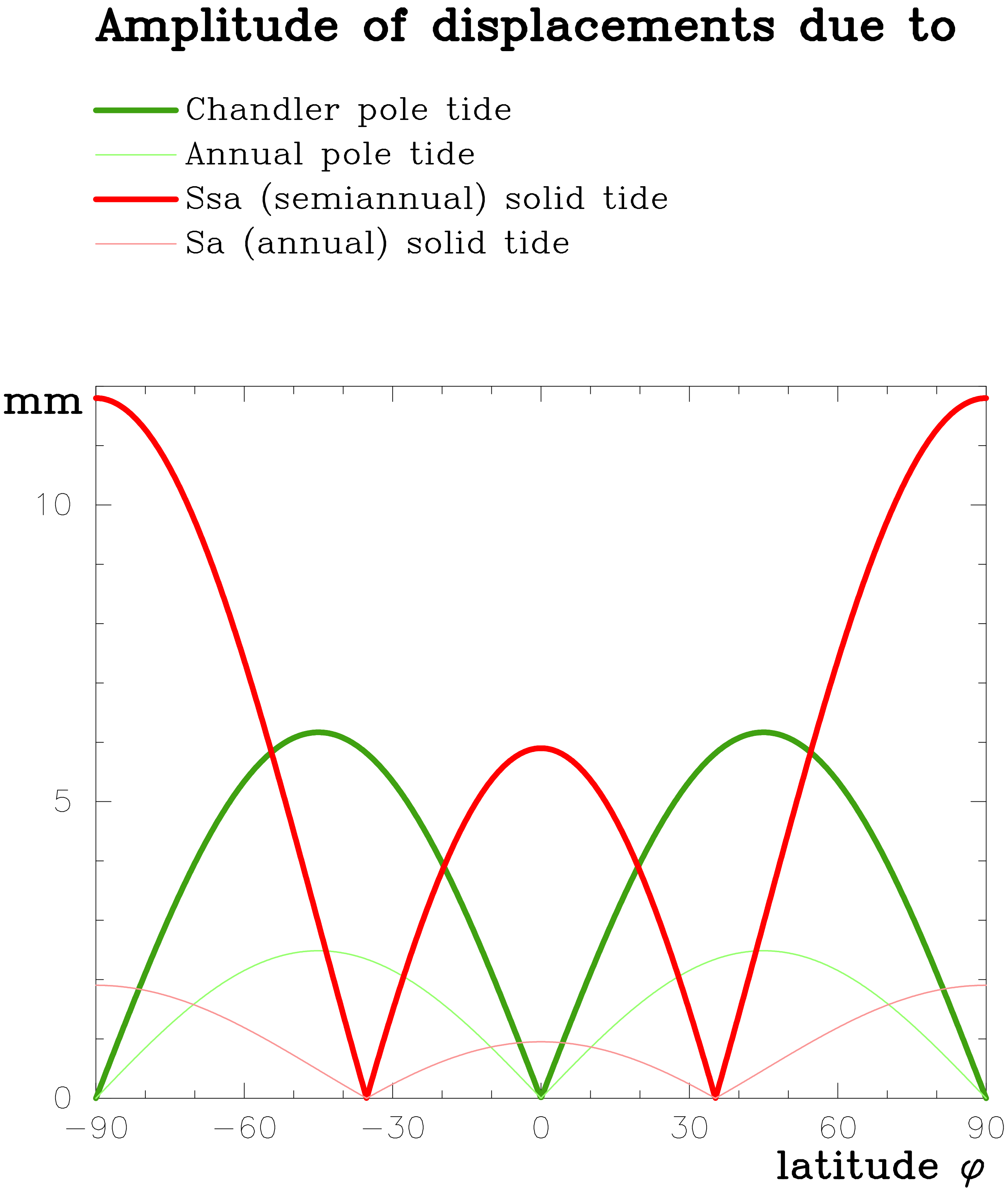
  - **A priori solid tide model**

    Abridged and simplified expressions of IERS 1996 Conventions were considered as inadequate for this task. I adhere to a purely harmonic approach.

    * Representation of the vector of tidal station displacement is taken from Mathews, (1997).
    * Functional dependence of Love numbers on frequency was restored from the tables in IERS 1996 Conventions. Variant with anelastisty was used.
    * HW95 harmonic expansion of TGP was used. 1024 terms were taken into account. Precision of computations is 50 microns.
    * Pole tide was presented as a sum of 1) shift, 2) linear drift; harmonic oscillation with 3) Chandler and 4) annual frequency, 5) remnant.

  - **Ocean loading**

    * CSR 3.0 model was used for 4 diurnal and 4 semidiurnal tides. Package SPOTL (D. Agnew, 1997) was used.
    * Alias tides within $\pm 4 \cdot 10^{-7}$ rad/sec wrt principal tides were taken into account. The same admittance as for the principal tides was assumed.
    * Ocean loading for Mm and Mf tides were computed using Egbert ocean models, ocean loading for Ssa and zonal tides with longer periods were computed under assumption of global equilibrium.
    * Ocean loading for annual and Chandler components of pole tide were computed under assumption of global equilibrium.



Amplitude of displacements due to
Chandler pole tide
Annual pole tide
Ssa (semiannual) solid tide
Sa (annual) solid tide
mm
10
5
0
−90
−60
−30
0
30
60
90
latitude φ

- **Model of estimation:**

  - Positions and velocities of primary 42 stations as global parameters;
  - Positions of primary 358 sources as global parameters;
  - Positions of secondary stations and secondary sources for each session;
  - Earth orientation parameters;
  - Clock and troposphere path delay modeled by linear spline with time span 1.0 and 0.5 hours respectively;
  - **Real and imaginary part of Love numbers h_2 and ℓ_2 at some frequencies**.

  No-net-translation, no-net-rotation constraints are imposed on positions and velocities of primary stations. No-net-rotation constraints are imposed on source positions.

  Different set of estimated Love numbers was tried.

- **Results:**

  – **Solution 1:** Zonal tides, pole tide;

Love numbers h_2 and ℓ_2:

pol	$h_2 = 0.518 \pm 0.015$	$\mathrm{Lag}(h_2) = 8^\circ \pm 2^\circ$
zon	$h_2 = 0.593 \pm 0.006$	$\mathrm{Lag}(h_2) = 1^\circ \pm 1^\circ$
pol	$\ell_2 = 0.082 \pm 0.004$	$\mathrm{Lag}(\ell_2) = -3^\circ \pm 2^\circ$
zon	$\ell_2 = 0.093 \pm 0.002$	$\mathrm{Lag}(\ell_2) = -3^\circ \pm 1^\circ$

  – **Solution 2:**

1. Zonal tides: 18.6 year, Sa, Ssa, Msm, Mm, Msf, Mf, Mstm, Mtm;
2. Pole tides: Chandler and Annual tides.

Love numbers h_2 and ℓ_2:

Cha	$h_2 = 0.63 \pm 0.02$	$\mathrm{Lag}(h_2) = -5^\circ \pm 1^\circ$
Ssa	$h_2 = 0.57 \pm 0.02$	$\mathrm{Lag}(h_2) = 15^\circ \pm 2^\circ$
Ann	$h_2 = 0.48 \pm 0.04$	$\mathrm{Lag}(h_2) = 74^\circ \pm 5^\circ$
Sa	$h_2 = 1.26 \pm 0.14$	$\mathrm{Lag}(h_2) = 41^\circ \pm 7^\circ$
Cha	$\ell_2 = 0.090 \pm 0.004$	$\mathrm{Lag}(\ell_2) = 8^\circ \pm 2^\circ$
Ssa	$\ell_2 = 0.109 \pm 0.004$	$\mathrm{Lag}(\ell_2) = -5^\circ \pm 2^\circ$
Ann	$\ell_2 = 0.112 \pm 0.010$	$\mathrm{Lag}(\ell_2) = -70^\circ \pm 7^\circ$
Sa	$\ell_2 = 0.21 \pm 0.03$	$\mathrm{Lag}(\ell_2) = 74^\circ \pm 9^\circ$

– **Solution 3:**

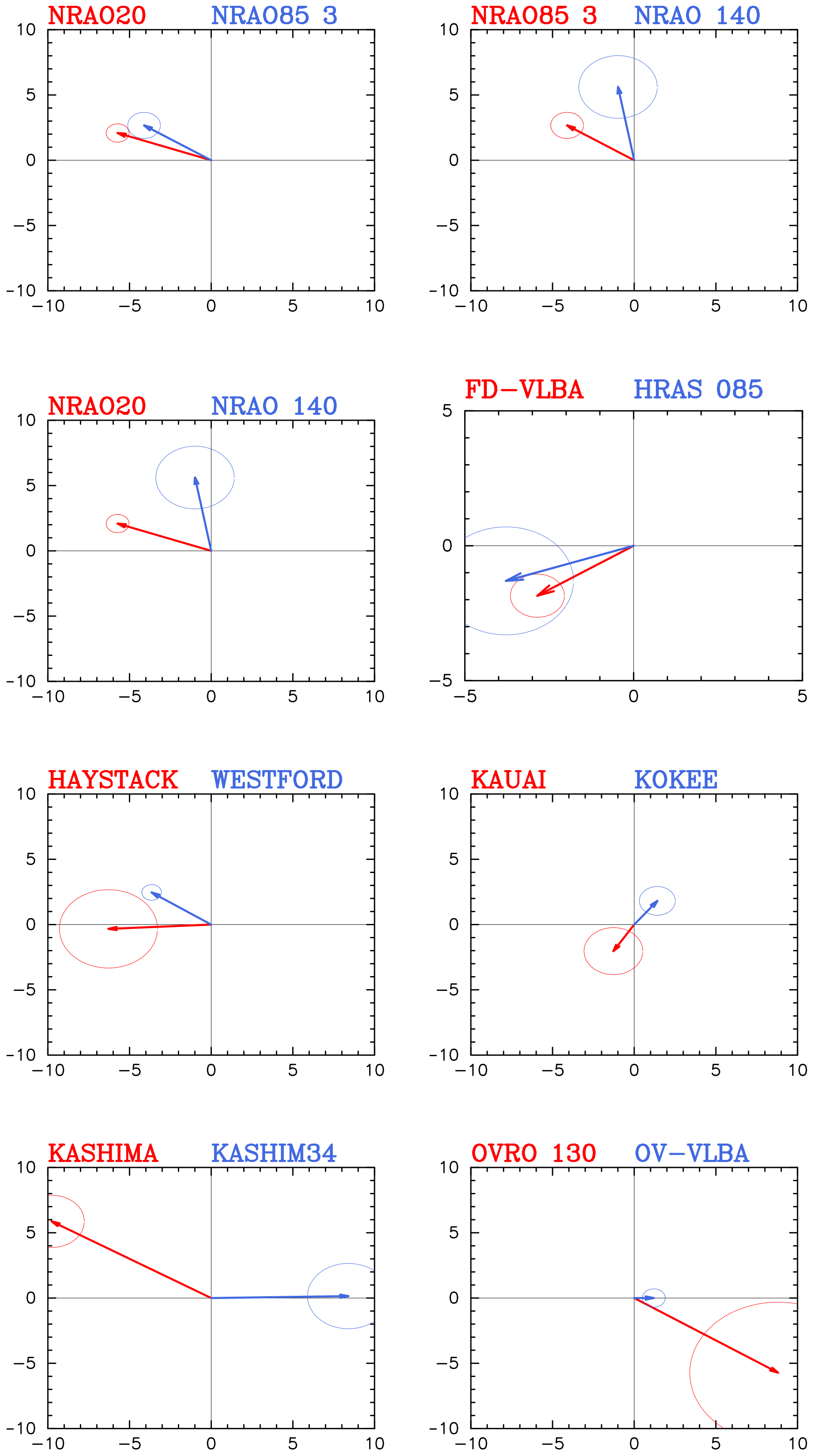
1. Zonal tide: 18.6 year, Ssa, Msm, Mm, Msf, Mf, Mstm, Mtm;
2. Pole tides: Chandler tides.
3. Annual variations of station positions.

Love numbers h_2 and ℓ_2:

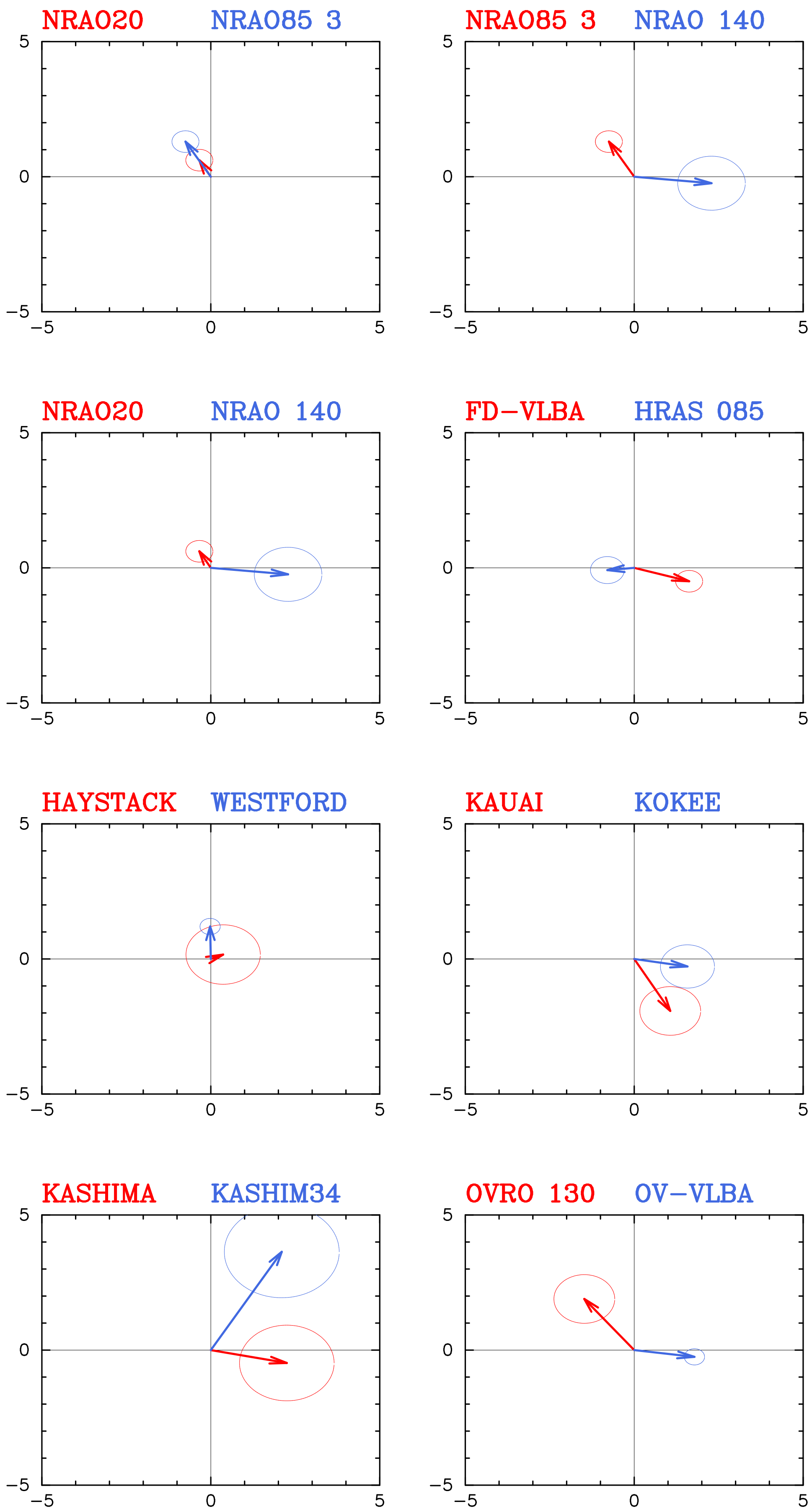
Cha	$h_2 = 0.61 \pm 0.02$	$\mathrm{Lag}(h_2) = 7° \pm 1°$
Ssa	$h_2 = 0.60 \pm 0.02$	$\mathrm{Lag}(h_2) = 16° \pm 2°$
Cha	$\ell_2 = 0.085 \pm 0.004$	$\mathrm{Lag}(\ell_2) = 9° \pm 2°$
Ssa	$\ell_2 = 0.105 \pm 0.004$	$\mathrm{Lag}(\ell_2) = -3° \pm 2°$

Amplitudes of annual station position variations are in the range 1–15 mm vertical and 1–7 mm horizontal.

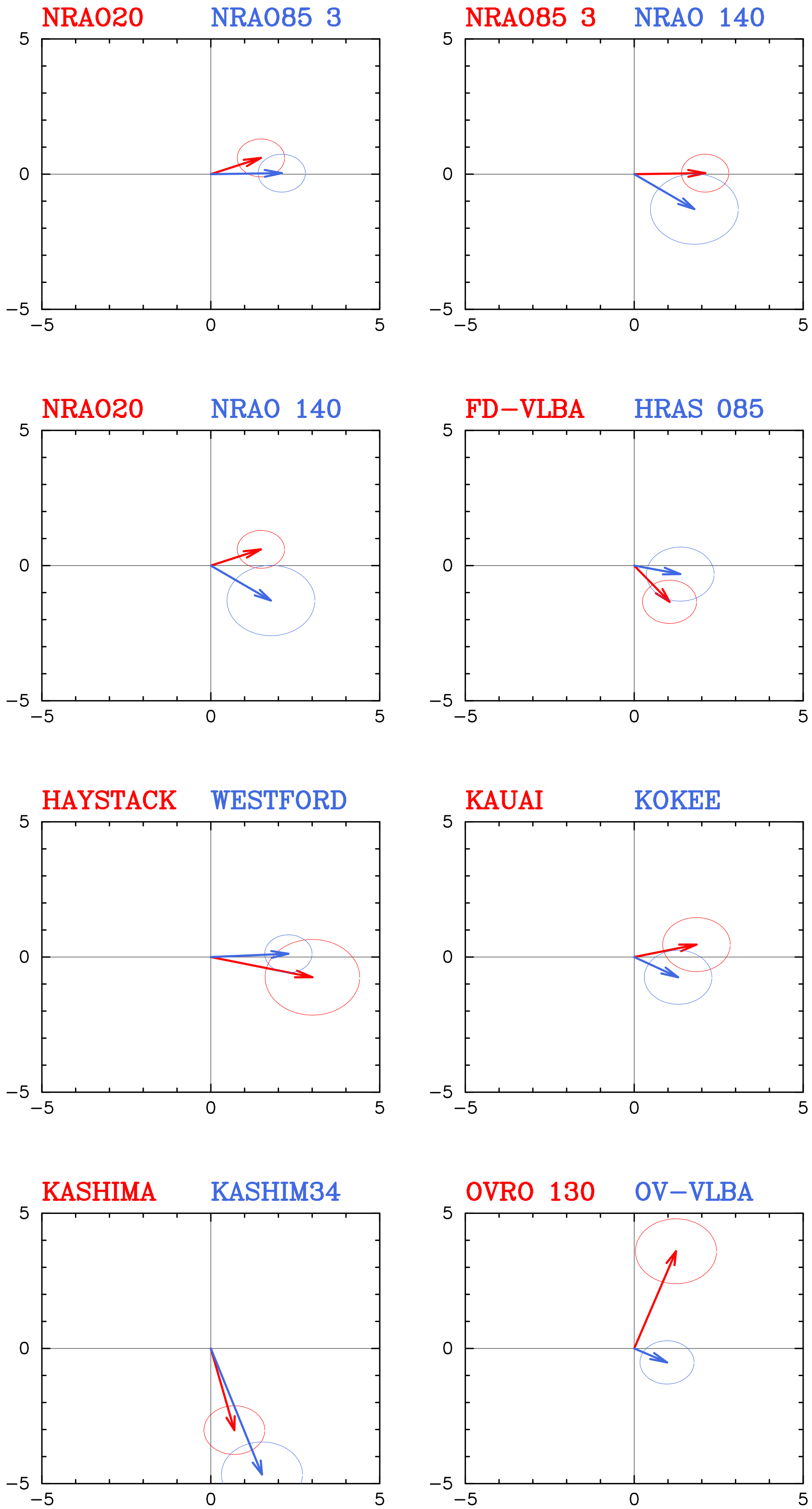
Phasors of VERTICAL annual positions variations of VLBI colocated sites (in mm)



Phasors of EAST annual positions variations of VLBI colocated sites (in mm)

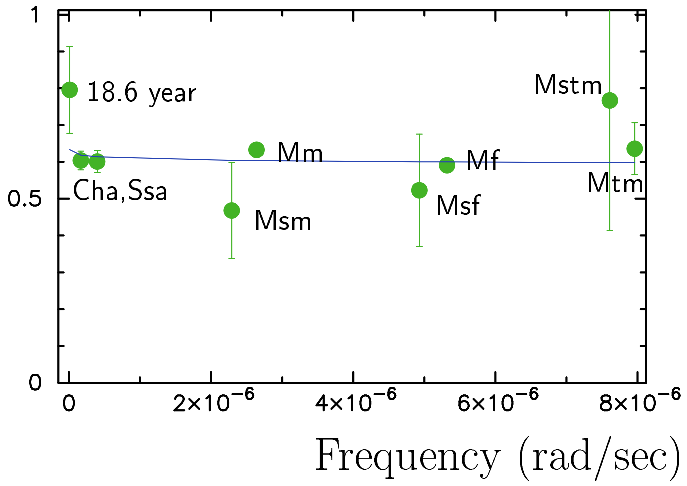


Phasors of NORTH annual positions variations of VLBI colocated sites (in mm)

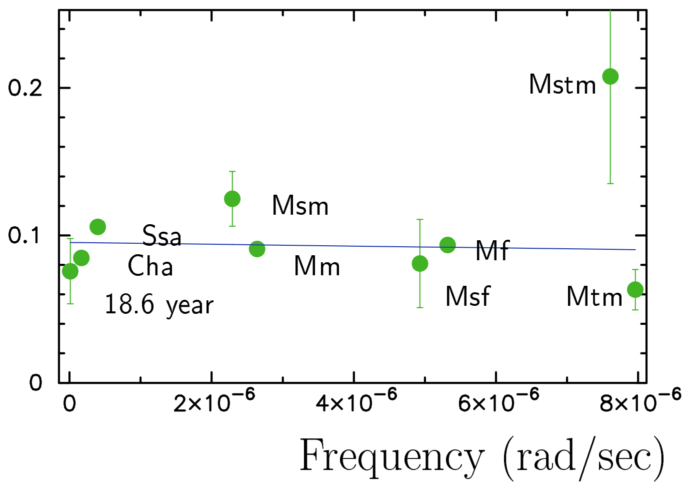


Estimates of Love numbers

h_2



ℓ_2



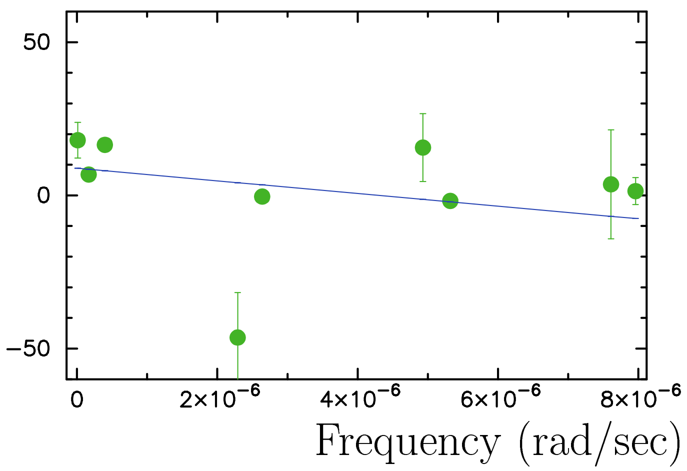
Fitting to a linear model: $n(f) = n_o + \alpha \log f/f_{\mathsf{Mf}}$

$h_2(\mathsf{Mf}) = \quad 0.60 \pm 0.01$	$\ell_2(\mathsf{Mf}) = \quad 0.092 \pm 0.004$
$\alpha(\mathsf{h}_2) \quad = -0.005 \pm 0.007$	$\alpha(\ell_2) \quad = -0.003 \pm 0.007$

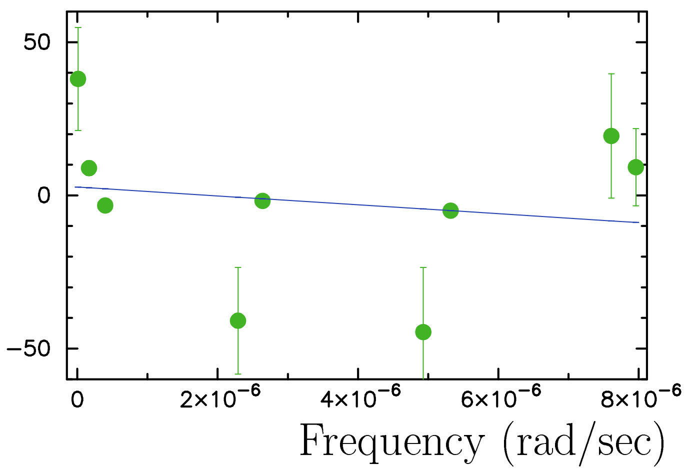
Formal uncertainties are scaled by $\sqrt{\chi^2/\mathrm{ndg}}$

Estimates of lag angles for Love numbers

Lag for h_2 (deg)



Lag for ℓ_2



- **Comparison with other results and theory**

L. Petrov (1995) pole tide:	$h_2 = 0.65 \pm 0.20$,	$\ell_2 = 0.11 \pm 0.05$
R. Haas and H. Schuh (1997) zonal tides:	$h_2 = 0.56 \pm 0.04$	$\ell_2 = 0.100 \pm 0.009$
J. Gipson, C. Ma (1998) pole tide:	$h_2 = 0.65 \pm 0.02$	$\ell_2 = 0.087 \pm 0.007$
This work, pole and zonal tides:	$h_2 = 0.60 \pm 0.01$	$\ell_2 = 0.092 \pm 0.004$
Theory: Mathews, (1997) elasticity model:	$h_2 = 0.600$	$\ell_2 = 0.083$
Theory: Mathews, (1997) anelasticity model:	$h_2 = 0.622$	$\ell_2 = 0.090$

- **Conclusions:**

  – The best estimates of Love numbers for long periods obtained by VLBI are

  $$h_2 = \quad 0.60 \pm 0.01 \qquad\qquad \ell_2 = \quad 0.092 \pm 0.004$$

  – Estimation of annual station position variations improves agreement between the estimates of Love number h_2 for Chandler pole tide and Ssa zonal tide.

  – Estimates of lag angle are not significantly different from 0 with uncertainty $\pm 4^\circ$ at 95% confidence level.

  – VLBI observations currently are unable to discriminate between theories of solid earth tides for long periods, although precision is approaching to the level when it would be possible.