ON OBSERVABILITY OF THE FREE CORE NUTATION

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ABSTRACT.

Neither astronomical technique, including VLBI, can measure nutation directly. Estimates of parameters of the nutation model are produced by solving the LSQ problem of adjusting millions parameters using estimates of group delay. The choice of the mathematical model for nutation used in the estimation process of analysis of group delays affects our ability to interpret the results. Ignoring these subtleties and using parameters of the nutation model either in the form of time series, or in the form of empirical expansion as "VLBI measurement of nutation", opens a room for misinterpretation and mistakes. Detailed analysis of the problem reveals that the separation of forced nutations, atmospheric nutations, ocean nutations, and the retrograde free core nutation requires invoking some hypotheses, and beyond a specific level becomes uncertain. This sets a limit of our ability to make an inference about the free core nutation.

1 INTRODUCTION

There are several basic misconceptions concerning the theory of nutation.

Misconception 1: *"VLBI measures nutation"*. In fact, VLBI measures ... the thermal noise at receivers with a tiny admixture of the noise from an observed extragalactic source. VLBI is the technique for the evaluation of the spectrum of the cross-correlation function using records of thermal noise synchronized by independent clocks. Using cross-spectrum, group delays can be estimated. The theory of wave propagation describes the dependence of group delay on the motion of the emitter and receivers, the properties of propagation media, and the phase fluctuations inside the electronic equipment. This dependence can be reduced to a parametric model, and parameters of this model can be adjusted using all available estimates of group delays.

Thus, nutation parameters are not measured, but estimated together with more than a million other parameters. Unlike to direct measurements, these estimates are heavily depends on a subjective choice of parameterization. One cannot adjust only nutation parameters — in that case the fit would be very poor. For this reason, one should not interpret nutation parameters alone, they have sense only as a part of the overall mathematical model that describes the motion of the station network.

Misconception 2: *"There exists a theory of forced nutation with accuracy comparable to observations"*. An elaborate theory of the non-rigid Earth nutation was developed by Wahr (1980) in 1970-s. It explained 91% of the deviation of the real Earth nutations from the absolute rigid body nutations. Numerical values of nutation expansion in the framework of this theory were computed using some integral quantities that depends on profiles of density and elasticity parameters inside the Earth. These profiles were derived from analysis of seismological data. However, the disagreement of Wahr's theory with observations is currently at a 500-σ level.

> *If it disagrees with experiment it is wrong. In that simple statement is the key to science. It does not make any difference how beautiful your guess is. It does not make any difference how smart you are, who made the guess, or what his name is — if it disagrees with experiment it is wrong. That is all there is to it.*

Feynman (1965), page 156.

Thus, we are compelled to acknowledge that the theory of nutation is wrong. Numerous attempts to improve the theory were undertaken, but they all failed. In recognition of this failure, some authors resorted to fitting parameters of their theories to the adjustments of nutation angles from VLBI analysis — just the quantities that the theory is supposed to predict. Certainly, by fitting a set of ad hoc parameters, one can get a set of coefficients of nutation expansion that may have whatever small fit to nutation angle adjustments. But this mathematical trick does not make the theory correct, and this set of coefficients cannot be called theoretical, but should be called empirical. Empirical expansions were presented in papers of Herring et al. (1986); Getino & Ferrandiz (2001); Shirai & Fukushima (2001); Mathews et al. (2002); Krasinsky & Vasilyev (2006).

The fact there is no precise theory of forced nutation, has important consequences. First, fitting parameters cannot be interpreted in terms consistent with the failed theory. Second, the residuals between the empirical theory and adjustments to nutation angles derived from VLBI analysis of group delay should not be interpreted as quantities with a specific physical meaning.

Misconception 3: *"An elaborate theory of nutation is needed for practical applications"*. Nutation has two major constituents: forced nutation with the precisely known excitation exerted by the Moon and the Sun and the constituents excited by the re-distribution of oceanic and atmospheric masses. The latter term is unpredictable in principle. The accuracy of determination of the nutation expansion from analysis of VLBI group delays has passed the level of atmospheric nutation contribution in 1990s. Therefore, even a precise theory of forced nutation would have been built, that theory would not be able to predict nutation with the accuracy comparable with observations. Similar to Chandler wobble, nutation parameters will have to be always determined from observations without any theory in mind.

2 DETERMINATION OF THE FREE CORE NUTATION

We consider here that N stations observe K celestial physical bodies. It is assumed that each station is associated with a reference point. In the case of VLBI antennas, this is the point of projection of the moving axis to the fixed axis. Observing stations receive electromagnetic radiation emitted by celestial bodies, and each sample of the received signal is associated with a time stamp from a local frequency standard synchronized with the GPS time. Analysis of voltage and time stamps of received radiation eventually allows us to derive the differences in the photon propagation time from observed bodies to reference points of observing stations. These distances depend on relative positions of stations with respect to observed bodies. The instantaneous coordinate vector of station i, in the inertial coordinate system at a given moment of time $\vec{r}_i^{\,\mathrm{C}}(t)$ is represented as the sum of a rotation and translation applied to a vector $\vec{r}_i^{\,\mathrm{T}}(t)$ in the terrestrial coordinate system as

$$\vec{r}_i^{\,\mathrm{C}}(t) = \widehat{\mathcal{M}}_a(t)\;\vec{r}_i^{\,\mathrm{T}} \;+\; \vec{q}_e(t) \times \vec{r}_i^{\,\mathrm{T}} \;+\; \vec{d}_i^{\,\mathrm{T}}(t) \;+\; \vec{T}(t) \qquad (1)$$

where $\widehat{\mathcal{M}}_a(t)$ is the a priori rotation matrix, $\vec{q}_e(t)$ is the vector of small rotation, $\vec{T}(t)$ is the translational motion of the network of stations, and $\vec{d}_i(t)$ is a displacement vector of an individual station. Equations of photon propagation tie the instantaneous vector of site coordinates $\vec{r}_i(t)$ with vectors of observed physical bodies and their time derivatives. These relationships allow us to build a system of equations of conditions. Solving these equations, we can get estimates of expansion of the vector $\vec{q}_e(t)$ over some basis functions.

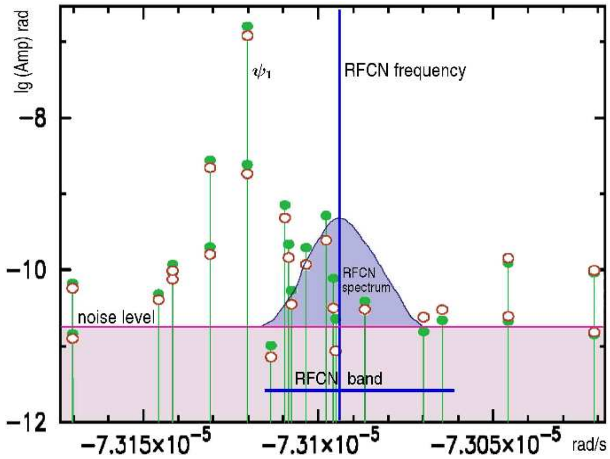
It has been demonstrated by Petrov (2007) that coefficients of expansions of the vector $\vec{q}_e(t)$ over the Fourier and B-spline bases can be found directly in a single LSQ solution that uses VLBI group delays estimates without resorting to intermediate time series. This means that we can estimate directly the spectrum of the Earth orientation variations from estimates of VLBI group delays. The portion of the spectrum of variations in q_1, q_2 considered as a complex process $q_1 + iq_2$ in the frequency range $[-1.5\Omega, -0.5\Omega]$, where Ω is the positive nominal frequency of the Earth rotation $7.292115146706979 \cdot 10^{-5}$ rad s^{-1}, is called nutation.

The nutation spectrum can be represented as a sum of two components: the forced nutation — a rail of with very sharp peaks with exactly known frequency, and the retrograde free core nutation (RFCN) — a band-limited continuous process with frequencies around the retrograde free core nutation frequency $-7.30901 \cdot 10^{-5}$ rad s^{-1}. Extensive discussion of nutation theory can be found in the monograph of Moritz (1987) and in the modern paper of Krasinsky (2006).

The problem of estimation of the free core nutation is reduced to the filtration of the observed spectrum of the Earth's rotation variations. The principle difficulty in separation of the RFCN from forced nutations is that their spectrum is overlapping, as figure 1 demonstrates.

We can separate the two constituents assuming 1) the RFCN is a band-limited process with known frequency band and unknown excitation; 2) the forced nutation is the purely harmonic process with known frequencies, known excitation, and a response function being *a smooth function* of frequency.

Figure 1: Spectrum of the free core nutations and forced nutations. Circles denote the power of the rigid Earth nutations, disks denote the power of the estimated nutation spectrum from VLBI group delays.



For practical implementation of this approach one should determine a) the band of the RFCN process and b) the empirical response function of forced nutations.

Empirical response function can be found with a data mining technique by comparing the spectrum of the Earth orientation with nutation constituents computed for the mechanical model of the absolutely rigid model. After a relatively short search, we find that complex amplitudes of forced nutations from VLBI data analysis A_e can be very well approximated to the theoretical amplitudes A_r for the mechanical model of the absolutely rigid Earth through this expression:

$$A_e(\omega) = [\alpha(\omega - \beta) + \frac{\gamma}{\omega - \delta}]\,(A_r(\omega) - A_n(\omega)) \tag{2}$$

where ω is the frequency, α, β, γ, and δ are complex parameters. A_n is the complex nutation amplitude caused by the non-tidal excitation, for instance, by the ocean and the atmosphere. It should be noted that functional dependence in the brackets of expression of 2, also called transfer function, can be derived analytically assuming 1) the triaxiality of the Earth's inertia ellipsoid is negligible; 2) the Earth consists of an elastic mantle and a liquid core; 3) the Earth's response is linear to the external torques. It should be stressed that although a simple theoretical considerations allow us to derive a parameterized expression similar to 2, the theory predicts *wrong* numerical coefficients, and therefore, it cannot be trusted.

Parameters α, β, γ, and δ can be determined from analysis of VLBI estimation results. We should be aware that 1) the estimates of coefficients α, β, γ, and δ heavily depend on a small set of constituents, including the constituent that corresponds to the ψ_1 tide, which is within the RFCN band; 2) estimation of the parameters of transfer function is a non-linear problem; 3) the oceanic and atmospheric contribution to the excitation has a significant uncertainty.

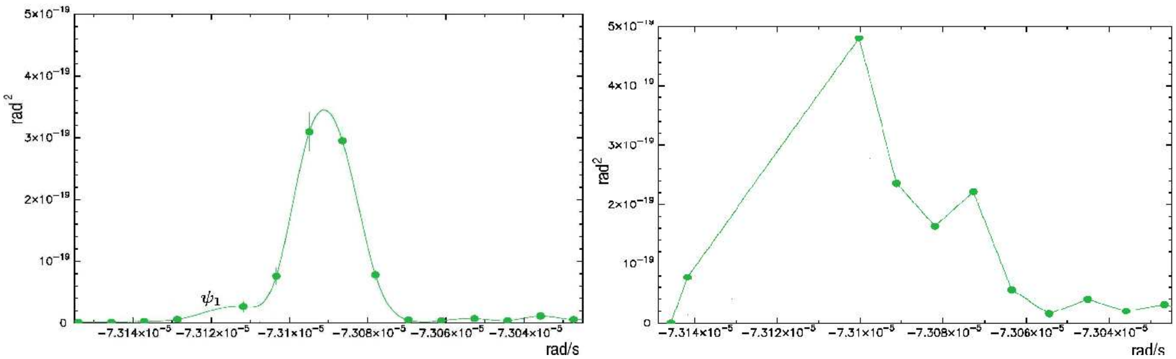
Using the estimates of the empirical transfer function, we can compute predicted forced nutations within the RFCN frequency band, and interpret the residual spectrum as the empirical RFCN spectrum. Predicted forced nutations have errors not only due to statistical uncertainties in the coefficients $\alpha, \beta, \gamma, \delta$, but also due to uncertainties in A_f and due to the errors of approximation in the expression 2. The fact that the spectrum of the RFCN and forced nutation is overlapping has important consequences: 1) the estimates of the RFCN spectrum have the statistical uncertainties due to the noise in VLBI group delay and the constraint uncertainties due to constituents separation, 2) estimates of the RFCN spectrum are not unique, but depend on assumptions made for parameterization of the empirical transfer functions; 3) estimates of the RFCN spectrum depend on the contribution of the ocean and the atmosphere on the main constituents of forced nutations.

Figure 2 shows an example of two estimates of the RFCN spectrum made under different assumptions for constituents separation. The estimation technique is described in details in (Petrov, 2007). It should be stressed that both spectra equally fit to VLBI group delays. The second spectrum shows a bi-modal pattern. We should resist to a temptation to rush inventing hypothesis for explaining this pattern.

In order to assess the sensitivity of the RFCN estimates to errors of oceanic and atmospheric contributions, I ran a Monte Carlo simulation. I ran 16 solutions and 1) added the Gaussian noise to estimates

Figure 2: Estimates of the RFCN power spectrum from VLBI time delays from 1984 through 2007 and resonance constraints on forced nutation. A) Empirical transfer function from ψ_1, K_1, P_1, O_1, etc was extrapolated to the entire RFCN band; B) Forced nutations within the RFCN band were ignored and as a result they propagated to the estimates of the RFCN spectrum.

A) B)



of nutations at K_1, S_1, P_1 due to uncertainty of ocean contribution with the standard deviation 150 prad; 2) added the Gaussian noise to estimates of nutations at ϕ_1, ψ_1 with the standard deviation 250 prad; 3) obtained new estimates of the empirical transfer function; 4) computed the new set of constraints on forced nutations. The added noise approximately corresponds to expected uncertainties of the oceanic and atmospheric contribution to nutation at these frequencies. Then I computed the rms of estimates of the RFCN spectra among these runs:

Uncertainty on the RFCN spectrum at different runs:	43 prad;
Formal uncertainty on RFCN from noise in group delays:	19 prad.

We see that constraint uncertainty is dominating.

3 CONCLUSIONS

It was shown that the estimates of the RFCN spectrum depend on the mathematical model used for separation of the free and forced nutations. The estimates are not unique. Separation of nutations results in a constraint uncertainty. The constraint uncertainty sets the limit of accuracy of the RFCN spectrum estimates and dominates the error budget. Since the RFCN has a continuous spectrum within its band, more precise observations will not result in improving of the accuracy of the RFCN spectrum determination.

References

Feynman R., "The Character of Natural Law", 1965, The MIT Press,

Getino J. & Ferrandiz J. M., 2001, "Forced nutations of a two-layer Earth model", MNRAS , 322, 785

Herring, T. A., Gwinn, C. R. & Shapiro, I. I., 1986, "Geodesy by radio interferometry: studies of the forced nutations of the earth. 1. Data analysis.", J. Geophys. Res., vol. 91, 4745

Krasinsky, G. A., 2006, "Numerical theory of rotation of the deformable Earth with the two-layer fluid core. Part 1: Mathematical model", Celes. Mech. Dyn. Astr. 96, 3–4, 169–217

Krasinsky, G. A., Vasilyev, M. V., 2006, "Numerical theory of rotation of the deformable Earth with the two-layer fluid core. Part 2: Fitting to VLBI data", Celes. Mech. Dyn. Astr. 96, 3–4, 219–237

Mathews P. M., Herring, T. A., & Buffett B. A., "Modeling of nutation and precession: new nutation series for nonrigid Earth an insights into the Earth's interior", 2002, J. Geophys. Res. 107, 10.1203/2001JB1000390

Moritz H., "Earth Rotation: Theory and Observation", 1987, Frederick Ungar, 617

Shirai, T., Fukushima, T., "Construction of a new forced nutation theory of the nonrigid Earth", 2001, AJ , 121, pp. 3270-3283.

Petrov L., "The empirical Earth rotation model from VLBI observations", A&A , vol. 467, p. 359, 2007.

Wahr, J. M., 1981, The forced nutations of an elliptical, rotating, elastic and oceanless Earth, Geophys. J. R. Ast. Soc., 64, 705