# Optimal estimation of the Earth orientation parameters using VLBI

*L. Petrov*

*Geodetic Institute of the University of Bonn*
*e-mail:* petrov@picasso.geod.uni-bonn.de

**Abstract**

The problem of the best strategy for estimation of the Earth orientation parameters using VLBI is considered. The present strategy for evaluation of 6 parameters for each session: UT1, UT1 rate, pole coordinates, nutation offsets contains internal contradictions and it has known deficiencies. An alternative parameterization is presented. The ways of its implementation and some results are discussed.

## 1. Introduction

Aims of the determination of the Earth orientation parameters (EOP) are a) to provide a numerical model to our consumers — to answer what was the Earth's orientation in the past and what will be the Earth orientation in the future; b) to improve our knowledge of the Earth interior — roughly speaking to measure deviations from the rotation of the absolutely solid body.

Two approaches for a description of a phenomenon of the Earth rotation are used: kinematic and dynamic. The kinematic approach deals with the question *how* the Earth is rotating. The dynamic approach answers the question what are *the reasons* of the variations in the Earth rotation. The kinematic approach considers constituents of the rotation as predictable/unpredictable. The dynamic approach considers them as deterministic/stochastic. As a result EOP are represented by 3 Euler angles as a function of time in the framework of the first approach and as a zoo of different components: precession, nutation, polhode, high-frequencies variations and so on in the framework of the second approach.

The choice of the approach inspires the certain strategy of parameters estimation. Two extremes are possible: 1) the theory describing the phenomenon is perfect and it is established up to a finite set of parameters; 2) no theory is established and the character of functional dependence is also a subject of evaluation. The more degrees of freedom — the less certain results. The more strict theory — the less chances to disclose new effects.

The dynamic approach is nearer to the first strategy of EOP estimation, while the kinematic approach assumes more freedom in parameterization.

The Earth rotation is a complicated process and there is no a parameterization which would be optimal for all problems and for all data sets. The choice of the best parameterization depends on the selected estimation strategy and on the properties of available data.

## 2. Strategy of EOP estimation

Constituents of the Earth rotation which are considered in the framework of the kinematic approach are summarized in the table 1.

Table 1. Constituents of the Earth rotation as reckoned from the rotating Earth

| | Component | Time scales | Frequencies | |
|---|---|---|---|---|
| 1 | Slowly variating terms | $> 2^d$ | $< 4 \cdot 10^{-5}$ | rad/sec |
| 2 | Harmonic components | $8^h$ – $18.^a6$ | $1 \cdot 10^{-8}$ – $2 \cdot 10^{-4}$ | rad/sec |
| 3 | High frequency aharmonic components | $8^h$ – $2^d$ | $4 \cdot 10^{-5}$ – $2 \cdot 10^{-4}$ | rad/sec |

A traditional strategy of EOP estimation implies a two-step procedure: a) splitting the dataset onto 24-hours segments (sessions) and adjustments of 6 parameters: $X_p$, $Y_p$, $UT1$, $\frac{\partial}{\partial t}UT1$, $\Delta\psi$, $\delta\epsilon$ for each session; b) analysis of the obtained EOP time series: filtering, LSQ fitting and etc.

Each coordinate of the polar motion is modeled by two parameters: a pole coordinate and a daily nutation offset. However, these parameters are separated perfectly only if duration of the session is exactly one Earth rotation cycle and changes of nutation angles during the session are negligible. In practice both these assumptions are violated, and the resulting series of pole coordinates and nutation angles are corrupted. This approach does not allow to estimate a constituent in the pole coordinates which has a frequency far away from the zero or diurnal frequency, for example semidiurnal polar motion. More important, the traditional method has an internal contradiction. If we assume that nutation is a purely harmonic process with known frequencies which corresponds to a quasi-diurnal polar motion then we need to estimates only amplitudes and phases of the harmonic components, but not to represent nutation angles as a time series. If we assume that there are aharmonic components in nutation, f.e. free core nutation, with a broad spectrum then we cannot strictly separate them from the pole coordinates.

I proposed an alternative way: a one-step approach. All parameters are determined in one global solution. The following parameters are estimated:

- Station positions and velocities, source coordinates.
- $X_p$, $Y_p$, $UT1$ and their rates of change for each session. They model slowly variating components.
- Set of sin- and cos- terms for $X_p$, $Y_p$ and $UT1$ at frequencies within the diurnal and semidiurnal bands (with respect to the rotating Earth). They include precession, CEP offset, nutations, tidal diurnal and semi-diurnal EOP variations, ad hoc harmonic variations. Frequencies are taken from a theory [1].
- Linear spline for $X_p$, $Y_p$ and $UT1$ with segment length 1.5 hours. These parameters model stochastic component in the Earth rotation.
- Nuisance parameters.

However, this choice of parameterization has some difficulties:

- Theoretical spectrum of EOP variations has multiplets: a set of components with near frequencies which cannot be resolved using the given data set.
- There are synonymous frequencies, f.e. nutation and diurnal high frequency variations in polar motion.
- Three constituents are overlapped and are not independent.
  - A session-average parameter and an average value of linear spline over the session are indistinguishable.
  - A session-rate parameter and a linear rate of change of linear spline over the session are indistinguishable.

- There is a quasi-linear dependence between a harmonic parameter and linear spline. The more number of nodes of the spline are taken the better a linear spline approximates a harmonic signal.

- There are "blind" frequencies due to sampling.

But we can overcome these difficulties.

- Set the lowest frequency. The harmonic components with frequencies which are nearer than the lowest frequency are grouped to multiplets. A common narrow-band amplitude and phase are estimated for all aliases. The assumption that ratios of amplitudes within a narrow band are known from a theory is used.
- Exclude synonymous frequencies.
- Impose stiff de-correlation constraints on linear spline for each session:
  - Average is zero (de-shifting);
  - Average rate is zero (de-trending);
  - Sin- and cos- components at the specified frequencies are zero (de-harmonization).

## 3. Results

The approach outlined above has been implemented in the CALC/SOLVE program package. All available VLBI observations for 1979.7–2000.0 years: $2.8 \cdot 10^6$ observations, 2961 sessions were used in a single combined solution.

736 harmonic parameters were estimated. They include parameters for precession, celestial pole offset, harmonic parameters at 148 nutation frequencies, 34 additional tidal frequencies and 3 ad hoc frequencies. The lowest frequency $1 \cdot 10^{-8}$ rad/sec (period 18.6 years) was used. The 2559 daily pole coordinates and UT1 as well as their rates and the 44150 parameters of linear spline for pole coordinates and UT1 were obtained.

Formal uncertainties of harmonic variations are 20-300 prad. The largest correlation is between the $18.^a6$ terms nutation and precession: 0.91. Only 89 correlations out of 270 480 are greater than 0.7. The estimates of stochastic components of EOP are shown in the figures 1–2.

Figure 1. Residual variations in the Earth rotation for a routine VLBI experiment

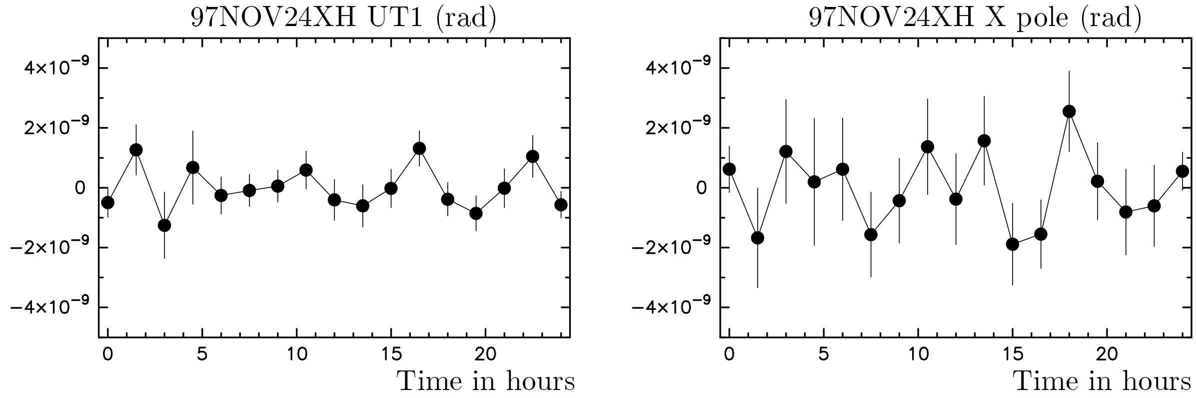
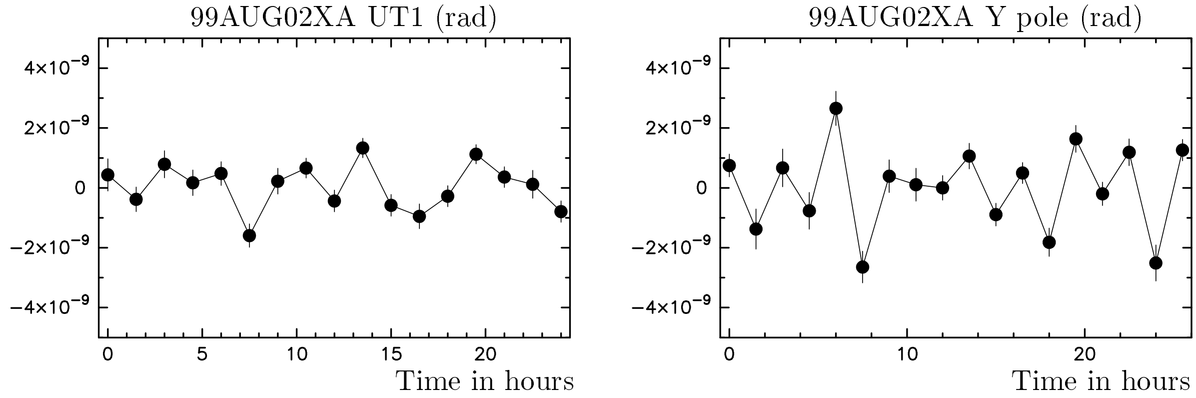


Figure 2. Residual variations in the Earth rotation for one of the best VLBI experiments




Results of comparison of the estimates of precession in longitude and obliquity rate are shown in the table 2. Comparison with the empirical nutation coefficients obtained by Souchay et al. [2] showed that the differences are within 1-2 nrad.

Table 2. Estimates of precession

| | | |
|---|---|---|
| Precession in longitude: | $-2.96 \pm 0.02$ mas/yr | (IERS96: $-2.96$ mas/yr) |
| Obliquity rate: | $-0.27 \pm 0.009$ mas/yr | (IERS96: $-0.23$ mas/yr) |

## 4. Conclusion

- It is shown that an empirical expansion of harmonic variations in the Earth rotation can be obtained in one step.
- Advantages of the proposed approach:
  - A uniform treatment of harmonic variations in the Earth rotation at diurnal and semi-diurnal frequencies;
  - No $\Delta\psi$, $\Delta\epsilon$ time series.
  - An ability to estimate correctly short-period nutation amplitudes.
- Disadvantage:
  - High computation cost.

## References

[1] J. Souchay, B. Loysel, H. Kinoshita, M. Folguera, “Correctiona and new developements in rigid earth nutation theory”, Astron. Astrophys. Suppl. Ser., vol. 135, pp. 111-131, 1999

[2] J. Souchay, M. Feissel, C. Bizouard, N. Capitane, M. Bougeard, “Precession and nutation for a non-rigid Earth: comparison between theory and VLBI observations”, Astron. Astrophys., vol. 299, p. 277-287, 1995