# Group delay ambiguity resolution algorithm

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23-DEC-97

Group delay observables are determined up to the arbitrary multiple of the certain constant as well as phase delay observables. This constant is reciprocal to the minimal spacing between channels in the band of synthesis and typically has value 50-200 nsec while phase delay ambiguity spacing has values about 120 psec for the X-band and 440 psec for S-band. It is reasonable first to master algorithms of resolving group delay ambiguities before wading in phase delay ambiguity resolution venture. In general resolving group delay ambiguities is rather more simple problem but at the same time both techniques contain some common elements.

SOLVE software didn't provide tools for resolving group delay ambiguities in automatic mode and allowed to do it only manually by pointing the cursor on the points of the plot and clicking mouse. It was a minor obstacle in analysis of the session with participation of 3–4 stations but became a substantial problem for analysis of recent sessions where 10-17 stations participated and required hours and hours of manual work.

Typical plots of the o-c for raw group delay are shown[1] in fig. ? for some the baselines for the IRIS-A session.

The following features are seen from the plots:

— o-c has jumps to be multiple of 100 nsec. It corresponds to minimal spacings between the channels in synthesis band to be equal to 10MHz what is typical frequency setup.

— o-c are grouped along the segments of straight lines. It stems from the fact that the clock function is the main contributor to the o-c, since usually we have good a priori values for all others parameters, but not for clocks. Contribution of clock shift to group delay is typically $10^{-5}$ – $10^{-6}$ sec, clock drift is $10^{-7}$ – $10^{-8}$, tropospheric delay is about $1–3 \cdot 10^{-9}$, while contribution of geodetic parameters is below $10^{-9}$ sec.

— There are points which deviate strongly from the set of other points. The magnitude of the deviation may have arbitrary values. These points indicate at incorrect work of appliance and should be discarded from solution.

It is essential that group delay ambiguity resolution algorithm should solve simultaneously four problems: determination of the ambiguity jumps for all points for both bands, outliers detection, determination preliminary values of clock function, determination group delay ionosphere correction. The following algorithm was implemented in SOLVE:

I. *Calculation ionosphere free linear combination of group delay observables.* Ionosphere contribution is typically 0.2–2.0 nsec for X-band and 3–30 nsec for S-band. Ionosphere free linear combination of group delay observables $\tau_g^{if}$ is used in analysis:

$$\tau_g^{if} = \tau_{gx}^o + \frac{f_{gs}^2}{f_{gx}^2 - f_{gs}^2}(\tau_{gx}^o - \tau_{gs}^o) \tag{1}$$

[1] Pictures are not available now.

We see that difference $(\tau^o_{gx} - \tau^o_{gs})$ is present in expression (1). The difference may be contaminated by ambiguities at both X- and S- bands what may lead to the jumps in ionosphere free combination of group delay observables. It is necessary to eliminate all these jumps. The following algorithm is proposed:

1. Cycle for baseline $j = 1, 2 \ldots$
   - 1.1. Determine "raw", contaminated by ambiguity jumps differences $(\tau^o_{gx} - \tau^o_{gs})$ for all observations.
   - 1.2. Divide the set of the differences onto K subsets by the manner that the deviation of each difference belonging to the K-th subset from the average over all the points from that subset don't exceed $C \cdot s$, where $C$ is minimal between ambiguity spacing constants for X- and S- bands, $s$ is some share of the spacing (recommended value: $s = 0.25$).
   - 1.3. Find the subset which contain the maximal number of points. The average difference over the points belonging to this subset is called as fiducial difference $d_F$
   - 1.4. Find ambiguity $N_i$ in the differences $d_i$ with respect to the fiducial difference $d_F$:

$$N_i = \text{Entier}\left(\frac{d_i - F_d}{C}\right) \qquad (2)$$

     where symbol Entier denotes operation of extraction integer part. This ambiguity is attributed to the X-band.

II. *Redistribution of fiducial differences between stations.* Since the fiducial differences in group delays for X- and S- bands were determined for all baselines independently the closures of triangles for three baselines may be violated. The fiducial differences for the baselines themselves were obtained only up to arbitrary ambiguity to be a multiple of $C$ (if to add *constant* $iC$ to all values of ambiguity for X-band and to add the same constant $iC$ to the baseline fiducial difference, the resulting differences remain the same). We should find correction of our estimates of fiducial differences in order to keep triangles of ionosphere contributions and therefore ionosphere free group delay observables closed. To do it we find station-dependent fiducial difference. The following algorithm was implemented:

1. Initialization. The attribute "not scanned" is set for all stations.
2. Cycle over all stations $i = 1, 2 \ldots$
   - 2.1. If the i-th station has been already scanned — go to the next station, else do the following:
   - 2.2. Take it as a reference station. Set station-dependent fiducial difference for the reference station equal to zero: $D_i := 0$.
   - 2.3. Set the attribute "scanned" for the i-th station.
   - 2.4. Cycle over all baselines: $j = 1, 2 \ldots$
     In dependence of the fact whether the stations k1 and k2 of the j-th baseline have been scanned already or not do the following:
   - 2.5. If the station k1 has been scanned but, the station k2 not, then determine fiducial difference for the k2-th station as $D_{k2} = D_{k1} + d_j$, where $d_j$ is fiducial difference for the j-th baseline. Set the attribute "scanned" for the k2-th station.
   - 2.6. If the station k2 has been scanned but, the station k1 not, then determine fiducial difference for the k1-th station as $D_{k1} = D_{k2} - d_j$, where $d_j$ is fiducial difference for the j-th baseline. Set the attribute "scanned" for the k1-th station.

2.7. Both stations were scanned. Calculate the residual error of the triangle closure for the fiducial difference of the j-th baseline $R_j$:

$$R_j = \text{Entier}\,\frac{(D_{k2} - D_{k1}) - d_j}{C}$$

Add $R_j$ to the value of fiducial difference for the j-th baseline: $d_j := d_j + R_j \cdot C$

3. Apply obtained correction to the fiducial differences to ambiguity jumps at X-band for all observation of all baselines: $N_j := N_j + R_j$

III. *Obtaining ionosphere free observables.* Only ionosphere free linear combination of group delay observables in according with expression (1) will be used starting from this moment. These ionosphere free observables don't have now jumps due to different ambiguities at X- and S-bands but may have jumps due to the same ambiguities for both bands.

IV. *Resolving group delay ambiguity for the observations of one baseline.* Divide entire set of observations onto subsets of the observations made at one baseline. Do the following operations.

1. Determine parameters of the linear trend for the phase delay rate. Delay rate observables have worse precision than group delays but they don't have ambiguities and therefore can be used to determine preliminary values of the parameters of one of the clocks for the baseline under consideration. o-c of delay rates can be modeled as $F = b + ct$. Coefficients found by LSQ fit of this model correspond to the following model of group delay: $\tau = a + bt + \frac{1}{2}ct^2$ under condition that all budget of o-c for both delay rates and group delays is determined by second order clock function. Thus, having determined parameters of the linear trend by fitting delay rates we find linear and quadratic term of clock function which influences on group delays. But we need to filter away outliers in delay rate observables which may distort considerably results of the fit. The following scheme has been implemented:

   1.1. All o-c for delay rates are scanned and those points not exceeding by modulo some threshold value take part in calculation of linear trend. If the share of not included points exceeds some limit then the threshold is increased twice and the operation is repeated.

   1.2. Points with residuals which deviate from the linear trend more then $3\sigma$ are barred from the solution and iterations are repeated while no outliers will be detected. Nevertheless, points marked as outliers for calculation of linear trend of delay rates will participate in further analysis.

2. Subtract linear and quadratic term from group delay observables.

3. Search of pairs of adjacent points with difference of o-c for group delay exceeding some constant. Both such points are temporarily barred from the solution.

4. Divide the set of observations of the baseline under consideration onto some segments by the manner that the maximal deviation from the average of o-c for each point belonging to the segment doesn't exceed certain specified constant $\tau_t$ (Recommended value $\tau_t = 8$ nsec) and observations in the segment don't lay so far from each other that the error of preliminary determination of clock linear and quadratic term would exceed some specified share of ambiguity spacing. The strategy of putting points into the same segment is to put the maximal points onto the same segment and at the same time to avoid putting there points with different ambiguities. To do it we scan array of o-c for group delays and make the following operations:

4.1. The first point is put into the first segment.

4.2. Each existing segment is tested whether the current point may belong to it or not. If $|\tau_i - \overline{\tau_s}| < \tau_t$ (where $\overline{\tau_s}$ — average over the s-th segment) then the i-th point is added to the s-th segment and the average is updated. New segment is created in the case when there is no one segment which would satisfy the condition above.

5. Find the segment which contains the maximal number of points. Average of o-c of group delay over the points from the segment yields us the first approximation for the clock shift for the current baseline. Ambiguities jumps are set to zero for all points from the segment.

6. Scan array of o-c for points out from the maximal segment and do the following:

6.1. Find the value of the ambiguity jump for the current observation:

$$J_i = \text{Entier}(\frac{\tau_i - \overline{\tau}}{S})$$

Where $S$ — group delay ambiguity spacing. In the case when $|\tau_i + J_i S - \overline{\tau}| > \tau_t$ the current observation is marked as outlier and excluded from further analysis.

6.2. Update of average: $\overline{\tau_{n+1}} := \dfrac{n\overline{\tau_n} + \tau_i}{n+1}$

V. *Redistribution permanent ambiguities between station clocks.* Having determined clocks function for all baselines we should find clock function for all stations except fiducial one. It is worth noting that the clock shifts found for all baselines have ambiguity to be a multiple of S (if to add *constant* $iS$ to all values of ambiguity jumps and to add the same constant $iS$ from baselines clock shift, the resulting o-c remain the same). This permanent ambiguity may lead to loss of triangle closure for the baseline clocks. We find correction to ambiguity of our estimates of clock shift in order to keep triangles closed. This correction will be taken into account when station-dependent clock function will be computed. The following effective algorithm was implemented. It is similar to the algorithm of redistribution fiducial differences between station.

1. Initialization. The attribute "not scanned" is set for all stations.

2. Cycle over all stations $i = 1, 2 \ldots$

2.1. If the i-th station has been already scanned — go to the next station, else do the following:

2.2. Take it as a fiducial station. Set clock shift for the fiducial station equal to zero: $A_i := 0$.

2.3. Set the attribute "scanned" for the i-th station.

2.4. Cycle over all baselines: $j = 1, 2 \ldots$
In dependence of the fact whether the stations k1 and k2 of the j-th baseline have been scanned already or not do the following:

2.5. If the station k1 has been scanned but, the station k2 not, then determine clock shift for the k2-th station as $A_{k2} = A_{k1} + a_j$, where $a_j$ is clock shift for the j-th baseline. Set the attribute "scanned" for the k2-th station.

2.6. If the station k2 has been scanned but, the station k1 not, then determine clock shift for the k1-th station as $A_{k1} = A_{k2} - a_j$, where $a_j$ is clock shift for the j-th baseline. Set the attribute "scanned" for the k1-th station.

2.7. Both stations were scanned. Calculate the residual error of the triangle closure for clock shift of the j-th baseline:

$$K_j = \text{Entier}\left(\frac{a_j - (A_{k2} - A_{k1})}{S}\right) S$$

$K_j$ has the meaning of permanent clock shift for the j-th baseline. Subtract it from the value of clock function for the j-th baseline: $a_j := a_j - K_j$

3. Correct ambiguity jumps for all observation of all baselines for permanent ambiguity of clock shit of the baseline: $J_j := J_j - K_j$